

REAL ANALYSIS - SYLLABUS

Program:

- Definition of Lebesgue outer measure: approximations by open sets, σ -subadditivity. Definition of measurable sets and Lebesgue measure: σ -algebra of Lebesgue measurable sets, Borel σ -algebra, σ -additivity and property of the measure, approximations by compact sets, construction of a nonmeasurable set. Measurable functions: definition, limits of measurable, approximation by simple functions, approximations by continuous functions, Egoroff and Lusin theorem
- Definition of Lebesgue integral: for simple functions, for bounded measurable functions (equivalence with the convergence of the series), for positive functions, for general functions. Comparison with Riemann integral. Convergence theorems: Fatou's Lemma, Monotone convergence (Beppo-Levi's Lemma), Lebesgue Convergence Theorem. Convergence in measure.
- Vitali covering lemma, differentiation of a monotone function, BV function in one space variable: total variation (positive, negative), differentiation a.e., Vitali/Devil staircase function. Differentiation of an integral, absolute continuity, fundamental theorem of calculus in the Lebesgue setting. Convex function, differentiation a.e., monotonicity of derivatives, Jensen inequality
- Baire theorem, categories. Measure theory in full generality, signed measures, Hahn decomposition, Radon-Nikodim, absolute continuity, orthogonal measures. Product measures, Fubini and Tonelli theorems

Text: Royden - Real Analysis