ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2011-12

Partial Differential Equations

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Real vector spaces with an inner product. Cauchy-Schwarz inequality.
Examples. Seminorms. Distances. Parallelogram identity.
Properties of l^2: completeness, separability, lackness of compactness and of sigma-compactness. Compactness of the Hilbert cube.

Hilbert spaces. Properties of the projection on a closed convex subset of a Hilbert space.
Orthogonal of a subspace. Linear operators on a Hilbert space: first properties.

Banach spaces. Examples. Norm of a linear operator.
The space of linear bounded operators between two Banach spaces. Topological dual of a Banach space.
Spaces of sequences. Dual of c_0. Dual of l_1.
The Riesz isometry for an Hilbert space.
Hamel bases. Schauder bases. Orthonormal systems. Parseval identity.
Examples. Fourier coefficients.

Hahn-Banach theorem: analytic form. Consequences.
Hahn-Banach theorem: geometric form. Consequences.
Kernels of linear operators. Separation of convex sets.

Banach-Steinhaus theorem. The open mapping theorem.

The space of test functions. Distributions. Order of a distribution.
Examples. Dirac's delta. Distributional derivatives. Examples.

Fundamental solution of the laplacian. Fundamental solution of the heat equation.

Support of a distribution. Convolution.

Fourier transform in L^1. Main properties of the Fourier transform.
Examples. Schwarz space. Fourier transform of a tempered distribution. Fourier transform in L^2. Sobolev spaces with real exponent.

Some application to linear elliptic partial differential equations. Weak solutions.