

ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2016-17

Partial Differential Equations

F. Maggi (10 lectures : 15 hrs)

The divergence theorem. Heat equation, Laplace equation, and their boundary value problems. Harmonic functions, laplacian in polar coordinates, the fundamental solution. Distributional Laplacian. Principle of superposition in electrostatics, representation formula for the Poisson equation and uniqueness. The heat kernel as a self-similar solution to the heat equation. The initial condition of the heat kernel and representation formula for the initial value problem on \mathbb{R}^n . Duhamel's principle and heat sources. Separation of variables for the heat equation on a bounded domain and the eigenvalue problem for the Dirichlet Laplacian. Distributional gradients and Sobolev spaces as Banach spaces. H^1 as an Hilbert space and the Dirichlet Laplacian as a self-adjoint operator on H^1 . Existence of solutions by means of the Poincare inequality and Riesz theorem. The Sobolev inequality on \mathbb{R}^n and proof of the Poincare inequality on a domain. The Dirichlet-Laplacian as a compact operator. Concentration and loss of compactness at the critical Sobolev exponent. Spectral theorem for compact self-adjoint operators on a Hilbert space.