

# **ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2015-16**

## **Partial Differential Equations**

G. Bellettini (10 lectures : 15 hrs)

Lecture 1, 6/4/2016.

Divergence theorem.

Dirichlet principle. Harmonic functions. Examples.

Lecture 2, 7/4/2016.

Fundamental solution of the laplacian.

Green's identity.

Lecture 3, 8/4/2016.

Surface mean values formula for harmonic functions.

Volume mean values formula for harmonic functions.

Lecture 4, 12/4/2016.

Volume mean value formula. The gradient estimate for harmonic functions.

Lecture 5, 13/4/2016.

Estimates of all derivatives. Analiticity of harmonic functions.

Limits of harmonic functions: the closure and the compactness theorem for harmonic functions.

Lecture 6, 14/4/2016.

Volume mean value formula for sub/super harmonic functions.

The weak maximum/minimum principles for sub/super harmonic functions.

The strong maximum/minimum principles for sub/super harmonic functions.

The strong loca maximum/minimum principles for harmonic functions.

The weak comparison principle.

Lecture 7, 20/4/2016.

Existence of minimizers of Dirichlet problem in  $\text{Lip}_k$ .

Super/sub minima.

Lecture 8, 21/4/2016.

Comparison principle for sub/super minimizers.

Exercise.

Lecture 9, 22/4/2016.

Existence of minimizers in  $\text{Lip}$ .

Lecture 10, 27/4/2016.

Poisson formula for the ball.