

ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2014-15

Partial Differential Equations

G. Bellettini (30 lectures : 45 hrs)

1. Harmonic functions

Classical harmonic and subharmonic functions.

The fundamental solution.

The surface mean value formula, and the volume mean value formula.

The gradient estimate for harmonic functions.

Analyticity of harmonic functions.

Weak and strong maximum/minimum principles.

The Harnack inequality.

Convergence of sequences of harmonic functions.

The Dirichlet problem on the ball: the Poisson kernel.

Elimination of singularities. The Hopf maximum principle.

The Perron's method: existence of the solution of the Dirichlet problem for continuous boundary data and bounded open sets satisfying an exterior ball condition.

2. Hilbert spaces and some functional analysis.

Properties of l^2 .

Compactness of the Hilbert cube.

Weak convergence in l^2 . Weak compactness of the unit ball.

A nonseparable Hilbert space.

Subspaces of a Hilbert space. Projection on a closed convex set. Orthogonality in Hilbert spaces. The Riesz representation theorem. Generalization to symmetric quadratic forms.

Sobolev spaces: weak derivative. Elementary properties and examples.

Density of smooth functions. The Banach-Steinhaus theorem and its corollaries.

Poincaré inequality. Dual of H^1_0 .

Adjoint operator of a bounded linear operator between Hilbert spaces.

Kernel and image of the adjoint operator.

Lax-Milgram theorem. Compact operators between Hilbert spaces.

The variational approach to elliptic problems in divergence form. The Fredholm theory in Hilbert spaces.

