ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2013-14

Partial Differential Equations

G. Bellettini (30 lectures : 45 hrs)

Wave equation in one space dimension. D'Alembert formula. Properties of solutions.

Linear transport equations. Method of characteristics. Distributional solutions. Geometric interpretations.

First order quasilinear equations: method of characteristics. Burgers' equation. Formation of a singularity. Geometric interpretation.
Comparison principle for smooth strictly convex first order conservation laws.

Nonlinear case: method of characteristics. Hamilton-jacobi equations.

The heat equation. Fundamental solution.

Weak comparison principle for smooth sub/supersolutions. Weak maximum/minimum principles. Uniqueness of solutions. Generalization to suitable linear parabolic operators.

Fundamental solution of the laplacian. Green's identities.
Representation theorem for C^2 functions in terms of the laplacian and normal derivatives, and in terms of the fundamentantal solution.
Poisson equation.

Real vector spaces with an inner product. Cauchy-Schwarz inequality.
Examples. Seminorms. Distances. Parallelogram identity.
Properties of l^2: completeness, separability, lackness of compactness. Compactness of the Hilbert cube.

Hilbert spaces. Properties of the projection on a closed convex subset of a Hilbert space.
Orthogonal of a subspace. Linear operators on a Hilbert space: first properties.

Banach spaces. Examples. Norm of a linear operator.
The space of linear bounded operators between two Banach spaces. Topological dual of a Banach space.
Spaces of sequences. Dual of c_0. Dual of l_1.
The Riesz isometry for an Hilbert space.

Hahn-Banach theorem: analytic form. Consequences.
Hahn-Banach theorem: geometric form. Consequences.
Kernels of linear operators. Separation of convex sets.

Banach-Steinhaus theorem. The open mapping theorem.

Definition of distribution: Dirac's delta.