ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2012-13

Partial Differential Equations

G. Bellettini (30 lectures : 45 hrs)

Examples of partial differential equations. Characteristic vectors. Characteristic hypersurfaces. Local solutions.

Linear transport equations. Method of characteristics. Geometric interpretation.

First order quasilinear equations: method of characteristics. Burgers' equation.
Formation of a singularity.
Comparison principle for smooth strictly convex first order conservation laws.

Nonlinear case: method of characteristics. Hamilton-jacobi equations.

Wave equation in one space dimension.
D'Alembert formula. Properties of solutions. Periodic solutions. Uniqueness.

The heat equation. Fundamental solution. Thyconoff example of non uniqueness. Weak comparison principle for smooth sub/supersolutions. Weak maximum/minimum principles. Uniqueness of solutions. Generalization to suitable linear parabolic operators.
Maximum principle on unbounded domains.

Fundamental solution of the laplacian. Green's identities.
Representation theorem for C^2 functions in terms of the laplacian and normal derivatives, and in terms of the fundamentantal solution.
Poisson equation.

Real vector spaces with an inner product. Cauchy-Schwarz inequality.
Examples. Seminorms. Distances. Parallelogram identity.

Properties of l^2: completeness, separability, lackness of compactness.
Compactness of the Hilbert cube.
Hilbert spaces. Properties of the projection on a closed convex subset of a Hilbert space.
Orthogonal of a subspace. Linear operators on a Hilbert space: first properties.

Banach spaces. Examples. Norm of a linear operator.
The space of linear bounded operators between two Banach spaces. Topological dual of a Banach space.
Spaces of sequences. Dual of c_0. Dual of l_1.
The Riesz isometry for an Hilbert space.

Hahn-Banach theorem: analytic form. Consequences.
Hahn-Banach theorem: geometric form. Consequences.
Kernels of linear operators. Separation of convex sets.

Banach-Steinhaus theorem. The open mapping theorem.

Definition of distribution: Dirac's delta.

First properties of the Fourier transform.