ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2015-16

Functional Analysis

G. Bellettini (20 lectures : 30 hrs)

Lecture 1, 3/2/2016.
Real inner products and real semi-inner products. Norms and
seminorms. Examples. Parallelogram identity. The parallelogram
identity characterizes norms induced by a scalar product.

Lecture 2, 4/2/2016.
Solutions to exercise given in lecture 1.
Gauge function of an open convex set containing the origin,
 of a normed space. Main properties.

Lecture 3, 4/2/2016.
l2 is separable, complete, uniformly convex. Initial discussion
on the properties of the unit ball.

Lecture 4, 10/2/2016.
Main properties of the Hilbert cube; compactness.

Lecture 5, 11/2/2016.
Wweak convergence in a separable Hilbert space.

Lecture 6, 12/2/2016.
The weak topology on l^2.
Example of a nonclosed subspace of l^2.
The unit ball of l^2 is weakly sequentially compact.

Lecture 7, 24/2/2016
The projection theorem on a convex closed set in Hilbert spaces.

Lecture 8, 25/2/2016
Applications of the projection theorem.

Lecture 9, 26/2/2016
Direct sum theorem. Riesz representation theorem.

Lecture 10, 2/3/2016
Hamel bases. Linear discontinuous functionals on a separable
Hilbert space. Separation of a closed convex set and a point outside.
A convex set is closed if and only if it is weakly closed.

Lecture 11, 3/3/2016
Baire's theorem. Norm of a linear bounded operator.

Lecture 12, 4/3/2016
Riesz representation theorem, Hahn-Banach theorem.

Lecture 13, 16/3/2016
adjoint operator in Hilbert spaces. Orthonormal systems. Existence
of a maximal orthonormal system. Relation with Hamel basis.

Lecture 14, 17/3/2016
Hilbert bases. Examples. Fourier expansion.

Lecture 15, 18/3/2016
Legendre polynomials. Bessel inequality. Parceval identity.

Lecture 16, 24/3/2016
Theorem of the Hilbert basis (also in the nonseparable case).
Examples of Hilbert bases.

Lecture 17, 25/3/2016
Self adjoint operators.
The spectral theorem in Hilbert
spaces for self-adjoint compact operators.

Lecture 18, 30/3/2016
The spectral theorem in Hilbert
spaces for self-adjoint compact operators.

Lecture 19, 31/3/2016
The spectral theorem in Hilbert
spaces for self-adjoint compact operators.
Abstract variational problems: the Lax-Milgram theorem.

Lecture 20, 1/4/2016
Abstract variational problems: the Lax-Milgram theorem,
the case of a symmetric bilinear form.
Introductory comments to Sobolev spaces.