

ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2018-2019

Functional Analysis

E. Carneiro (20 lectures :30 hrs)

Syllabus:

- 1) Review of real analysis: Normed vector spaces; Banach spaces; L^p -spaces; Convolutions and approximations of the identity; Young's inequality; Hardy-Littlewood maximal function and Lebesgue differentiation theorem.
- 2) Linear operators; Adjoint; Hahn-Banach theorem; Baire category theorem; Uniform boundedness principle (Banach-Steinhaus theorem); Closed graph theorem; Open mapping theorem.
- 3) Weak and weak* topologies; Banach-Alaoglu theorem; Kakutani's theorem; Mazur's lemma; Reflexive spaces; Separable spaces; Uniformly convex spaces.
- 4) Hilbert spaces; Orthonormal sets and bases; Riesz representation theorem; Lax-Milgram theorem; Reproducing kernels.

Additional topics (time permitting):

- 5) Fourier transform; Riemann-Lebesgue lemma; Plancherel's theorem; Hausdorff-Young inequality; Schwartz class; Tempered distributions; Fourier series; Poisson summation.
- 6) Compact operators; Spectral theory of compact self-adjoint operators; Fredholm's alternative.
- 7) Sobolev spaces; Approximations of Sobolev functions; Glagliardo-Nirenberg-Sobolev inequality; Sobolev embeddings; Weak solutions for elliptic PDEs; Existence of weak solutions.

Reference books:

- 1) [Brezis, Haim](#) Functional analysis, Sobolev spaces and partial differential equations. [Universitext](#). Springer, New York, 2011
- 2) [Folland, Gerald B.](#) Real analysis. Modern techniques and their applications. Second edition. [Pure and Applied Mathematics \(New York\)](#). A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1999.
- 3) [Stein, Elias M.](#); [Weiss, Guido](#) Introduction to Fourier analysis on Euclidean spaces. Princeton Mathematical Series, No. 32. Princeton University Press, Princeton, N.J., 1971.
- 4) [Stein, Elias M.](#); [Shakarchi, Rami](#) Real analysis. Measure theory, integration, and Hilbert spaces. [Princeton Lectures in Analysis, 3](#). Princeton University Press, Princeton, NJ, 2005.