ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2017-18

Functional Analysis

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Seminorms and norms on a real or complex vector space.
Examples: L^p, C(K). Space of sequences.
Strict convexity and uniform convexity. Semi-inner products, and inner products. The parallelogram identity. Characterization of norms coming from an inner product. Uniform convexity of a norm coming from an inner product.
Algebraic dual and bidual. The algebraic canonical mapping and its injectivity. Axiom of choice and Zorn lemma.
Topological dual. Norm of a linear bounded functional.
Main properties of the dual norm.
Hamel basis: existence, and algebraic dimension of a vector space.
Existence of a linear unbounded functional on a vector space.
Topological dual space. Definition of Banach space. On a Banach space of not finite dimension, a Hamel base cannot be countable.
Dual of c_0, and of l^1. Hahn-Banach theorem: analytic form and consequences. The canonical embedding. Exercises on l^2. The Hahn-Banach separation theorems.
Weak sequential compactness of the unit ball of l^2.
The Hilbert cube. Weak-star topology and weak convergence.
Separability properties. The unit ball in the dual of a Banach space is weakly* compact.
The Banach-Steinhaus theorem. Weakly* bounded sets are bounded.
Baire's theorem. Projection on a closed subspace in a Hilbert space.
Biorthogonal. Topological direct sum of a Hilbert space as a closed subset plus its orthogonal. Characterization of the point of minimal distace from a closed subspace using the ``Euler'' equation.
Projection on a closed convex subset of a Hilbert space.
Characterization using the ``Euler'' inequality.
The Riesz representation theorem. Adjoint operator.
Othonormal systems. Maximal orthonormal systems. Complete orthonormal systems. Hermite polynomials.
Fourier expansions. The Hilbert base theorem in the separable case.
Characterization of separable Hilbert spaces.