ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2018-2019

Complex Analysis

Fernando Rodriguez Villegas (15 lectures: 22.5 hrs)

Syllabus - 2018

(1) Review of complex numbers: plane representation, polar coordinates, conjugation. Basic topology, Euclidean plane.
(2) Complex differentiation, Cauchy-Riemann equation. Riemann stereographic projection.
(3) Cauchy sequences, series, Weierstrass M-test. Zeros and poles. Rational functions, equal number of zeros an poles in the Riemann sphere. Möbius transformations.
(4) Power series, radius of convergence, analyticity. Product of series. The exponential, cosine, sine and logarithm functions.
(5) Analytic functions as maps. Conformal maps. Möbius transformations, basic hyperbolic geometry in the plane.
(6) Path integrals. Cauchy's theorem. Goursat's proof for a triangle. Winding number of a path. Path homologous to zero, simply connected regions.
(7) General Cauchy's theorem, Cauchy's integral formula, Cauchy's estimate. Liouville's theorem. Fundamental theorem of algebra. Morera's theorem.
(8) Removable singularities. Power series expansion, examples. Uniqueness of analytic functions. Casorati-Weierstrass theorem.
(9) Local mapping properties of analytic functions. Maximum principle. Schwarz lemma.
(10) Residue theorem. Argument principle. Rouché's theorem, applications to zeros of polynomials. Gershgorin's lemma.
(11) Jordan's lemma. Calculation of real integrals by residues.
(12) Laurent expansion. Infinite products. Weierstrass theorem on convergence of analytic functions.
(13) Basic theory of the Gamma function.