

ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2012-13

Complex Analysis

F. Vlacci (20 lectures : 30 hrs)

Introduction to the field of Complex numbers. Gauss plane and polar representation of complex numbers. The extended plane (Riemann sphere): its spherical representation, topological and metric properties.

Complex power series: absolute/uniform convergence and radius of convergence.

Hadamard's

Criterion for convergence. Holomorphicity and complex analyticity: Cauchy–Riemann equations and elementary properties of holomorphic functions. Abel's Lemma.

Complex integration (mainly along paths). Index of a closed curve. Cauchy's Integral Formula.

Power series representation of holomorphic functions. Cauchy's Estimates.

Zeros of holomorphic functions: counting multiplicities and Rouché's Theorem. The Fundamental Theorem of Algebra. Open mapping theorem and the Maximum Modulus Theorem.

Morera's Theorem.

Conformality of holomorphic functions. Cayley Transformation.

Classification of singularities. Weierstrass–Casorati Theorem. Residues and the Residue Theorem. The Argument Principle. Meromorphic functions.

Schwarz's Lemma and Schwarz–Pick's Lemma. Möbius transformations. Classifications of automorphisms of the unit disc, complex plane and Riemann sphere.

Convergence of holomorphic functions: Weierstrass' Theorem and Hurwitz' Theorem.

Runge's

Theorem and Mittag Leffler Theorem (only statements). Normal families. Montel's Theorem.

Riemann Mapping Theorem. Analytic Continuation.

Introduction of elements of hypercomplex analysis over quaternions, Cayley numbers and Clifford Algebras.

