ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2011-12

Complex Analysis

F. Vlacci (20 lectures : 30 hrs)

Introduction to the field of Complex numbers. Gauss plane and polar representaion of complex numbers. The extended plane (Riemann sphere): its spherical representation, topological and metric properties.
Complex power series: absolute/uniforme convegence and radius of convergence. Hadamard's Criterion for convergence. Holomorphicity and complex analiticity: Cauchy–Riemann equations and elementary properties of holomorphic functions. Abel's Lemma.
Complex integration (mainly along paths). Index of a closed curve. Cauchy's Integral Formula.
Power series representation of holomorphic functions. Cauchy's Estimates.
Zeroes of holomorphic functions: counting multiplicities and Rouche's Theorem. The Fundamental Theorem of Algebra. Open mapping theorem and the Maximum Modulus Theorem.
Morera's Theorem.
Conformality of holomorphic functions. Cayley Transformation.
Classification of singularities. Weierstrass–Casorati Theorem. Residues and the Residue Theorem. The Argument Principle. Meromorphic functions.
Schwarz's Lemma and Schwarz–Pick's Lemma. Möbius tranformations. Classifications of automorphisms of the unit disc, complex plane and Riemann sphere.
Convergence of holomorphic functions: Weierstrass' Theorem and Hurwitz' Theorem.
Runge's Theorem and Mittag Leffler Theorem (only statements). Normal families. Montel's Theorem.
Riemann Mapping Theorem. Analytic Continuation.
Introduction to Riemann Surfaces as quotient spaces: universal covering and subgrous of au-tomorphisms whose actions are free and properly discontinuous. Classification of Riemann Surfaces and Moduli Space of Complex tori.