

ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2018-2019

Algebraic Topology

P. Putrov (10 lectures : 15 hrs)

Syllabus:

- Homotopy equivalence
- (Deformation) retract
- Paths in a topological space, operations on paths, path homotopy.
- Fundamental group π_1 : definition, group structure, independence of the base point
- Covering space, lifting property
- $\pi_1(S^1)$, calculation and applications (including Brouwer fixed point theorem)
- Pushforward map on π_1 , its properties.
- Homotopy invariance of π_1 .
- $\pi_1(X \times Y)$.
- van Kampen's Theorem, applications (including $\pi_1(S^n)$, $\pi_1(X \vee Y)$)
- Simplicial homology, examples
- Singular homology: definition, basic properties, homology of a point
- Induced map between chain complexes, homologies (pushforward). Homotopy invariance of pushforward, chain homotopy.
- Relative homology, long exact sequence property, induced maps
- Excision theorem, homology of a quotient, applications (homology of a sphere, invariance of the dimension)

Recommended literature:

- Hatcher "Algebraic Topology", <http://pi.math.cornell.edu/~hatcher/AT/AT.pdf>
- Matveev, "Lectures on Algebraic Topology"