

ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2014-15

Algebra

L. Göttsche (20 lectures : 30 hrs)

(A) Groups

Definitions and examples of: group, homomorphism, kernel image, normal subgroup, quotient subgroup (equivalence relations in general), cyclic groups, action of a group on a set, basic results in the symmetric group, conjugacy classes, Sylow theorems and applications.

(B) Rings:

Definition of ring, first properties and examples. Zero divisors, integral domains, units of a ring and fields.

Homomorphisms of rings, ideals, quotient rings, homomorphism theorem.

Prime ideals; maximal ideals.

Polynomial rings, Euclidean algorithm (division with rest), Euclidean rings, principal ideal domains.

Irreducible elements, Eisenstein's criterion for irreducibility of polynomials.

(B) Fields:

Characteristic, field extensions, degree theorem, algebraic extensions and simple algebraic extensions, algebraic closure, splitting fields. Normal and separable extensions, finite fields, Galois groups, fundamental theorem of Galois theory, cubic polynomials. Solution of polynomials by radicals

References:

M. Artin: Algebra I.N. Herstein: Topics in Algebra. Lecture notes will be provided.