

ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2015-16

Algebra

L. Göttsche (20 lectures : 30 hrs)

(A) Groups

Definitions and examples of: group, homomorphism, kernel image, normal subgroup, quotient subgroup (equivalence relations in general), cyclic groups, action of a group on a set, basic results in the symmetric group, conjugacy classes, Sylow theorems and applications.

(B) Rings:

Definition of ring. Integral domains and fields.

Homomorphisms of rings, ideals, quotient rings, homomorphism theorem.

Prime ideals; maximal ideals.

Polynomial rings, Euclidean algorithm, Euclidean rings, principal ideal domains.

Irreducible elements, criteria for irreducibility of polynomials.

(B) Fields:

Characteristic, field extensions, degree theorem, algebraic extensions and simple algebraic extensions, algebraic closure, splitting fields. Normal and separable extensions, theorem of the primitive element, finite fields, Galois groups, fundamental theorem of Galois theory, cubic extensions, cyclotomic extensions, solution of polynomials by radicals.

References:

M. Artin: Algebra I.N. Herstein: Topics in Algebra. Lecture notes will be provided.