ICTP DIPLOMA PROGRAMME IN MATHEMATICS 2013-14

Abstract Algebra

L. Göttsche (20 lectures : 30 hrs)

(A) Groups
Definitions and examples of: group, homomorphism, kernel image, normal
subgroup, quotient subgroup (equivalence relations in general), cyclic
groups, action of a group on a set, basic results in the symmetric group, conjugacy
classes, Sylow theorems and applications.

(B) Rings:
Definition of ring, first properties and examples. Zero divisors, integral domains, units of
a ring and fields.
Homomorphisms of rings, ideals, quotient rings, homomorphism theorem.
Prime ideals; maximal ideals.
Polynomial rings, Euclidean algorithm (division with rest), Euclidean rings, principal
ideal domains.
Irreducible elements, Eisenstein's criterion for irreducibility of polynomials.

(B) Fields:
Characteristic, field extensions, degree theorem, algebraic extensions and simple
algebraic extensions, algebraic closure, splitting fields. Normal and separable
extensions, finite fields, Galois groups, fundamental theorem of Galois theory, cubic
polynomials. Solution of polynomials by radicals

References:
M. Artin: Algebra I.N. Herstein: Topics in Algebra. Lecture notes will be provided.