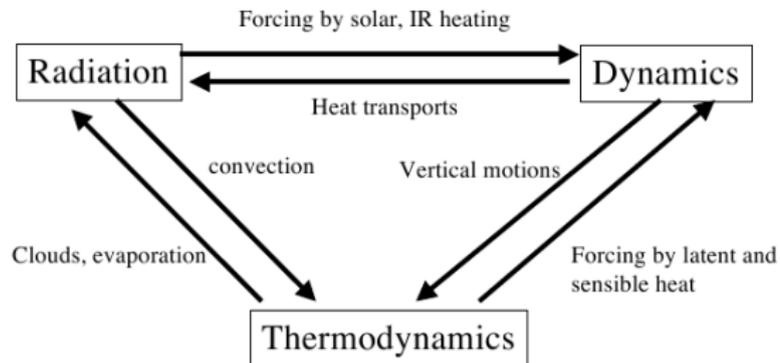


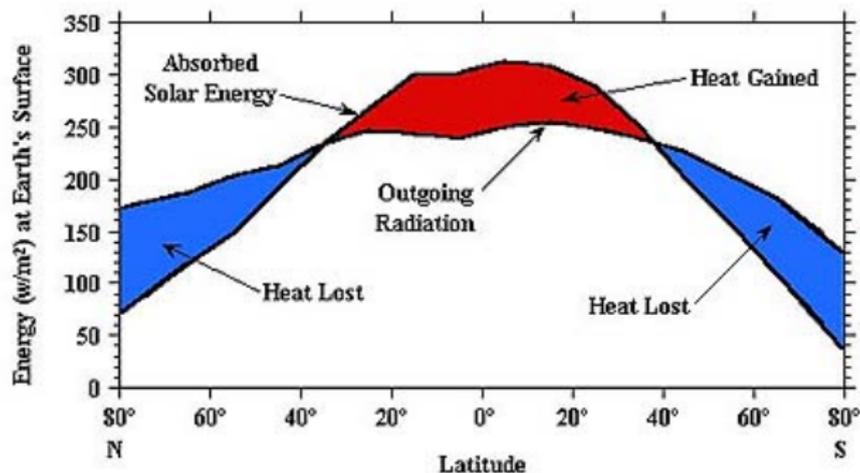
# Course overview



## This course will concern itself with

- Thermodynamics
- Cloud Physics
- Radiation

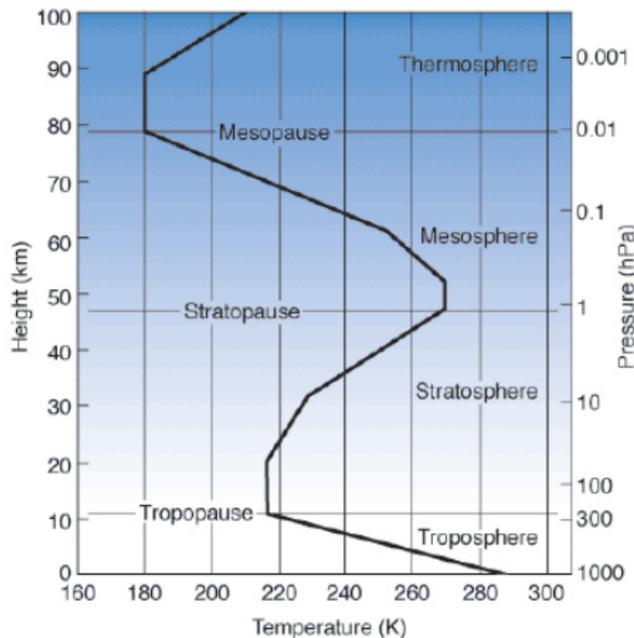
# Net Radiation Budget



Solar radiation absorbed by earth-atmosphere system balanced by outgoing thermal radiation on average globally, but not locally.

**Q: How is a local energy equilibrium maintained?**

# Atmospheric vertical structure



**Atmosphere is divided into distinct vertical layers according to temperature lapse-rate:**

- Lower (troposphere)
- middle (stratosphere and mesosphere)
- upper (thermosphere)

This course is concerned with tropospheric processes

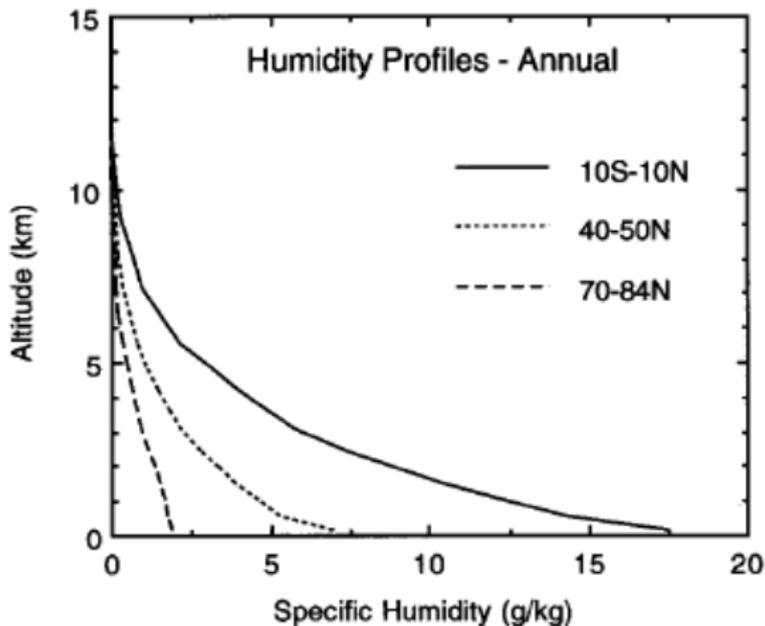
# The atmospheric gases

Constituent	Mol. Wt.	Conc. by vol.	
Nitrogen (N <sub>2</sub> )	28.013	0.7808	} Major constituents
Oxygen (O <sub>2</sub> )	32.000	0.2095	
Argon (Ar)	39.95	0.0093	
Water vapor (H <sub>2</sub> O)	18.02	variable	} Minor constituents
Carbon dioxide (CO <sub>2</sub> )	44.01	380 ppmv	
Neon (Ne)	20.18	18	
Helium (He)	4.00	5	
Methane (CH <sub>4</sub> )	16.	1.75 "	
Hydrogen (H <sub>2</sub> )	2.02	0.5 "	
Nitrous oxide (N <sub>2</sub> O)	56.03	0.3 "	
Ozone (O <sub>3</sub> )	48.00	0-0.1 "	

Q:

Are these gases well-mixed in the atmosphere?

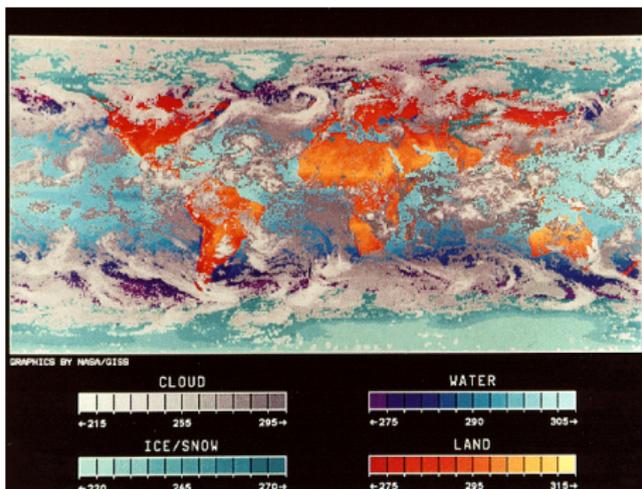
# Atmospheric Water Vapour Distribution



It is seen that the concentration of water vapour in the atmosphere falls rapidly with height, and is much higher in the tropics relative to the subtropics and midlatitudes.

Q: Do you know why this might be?

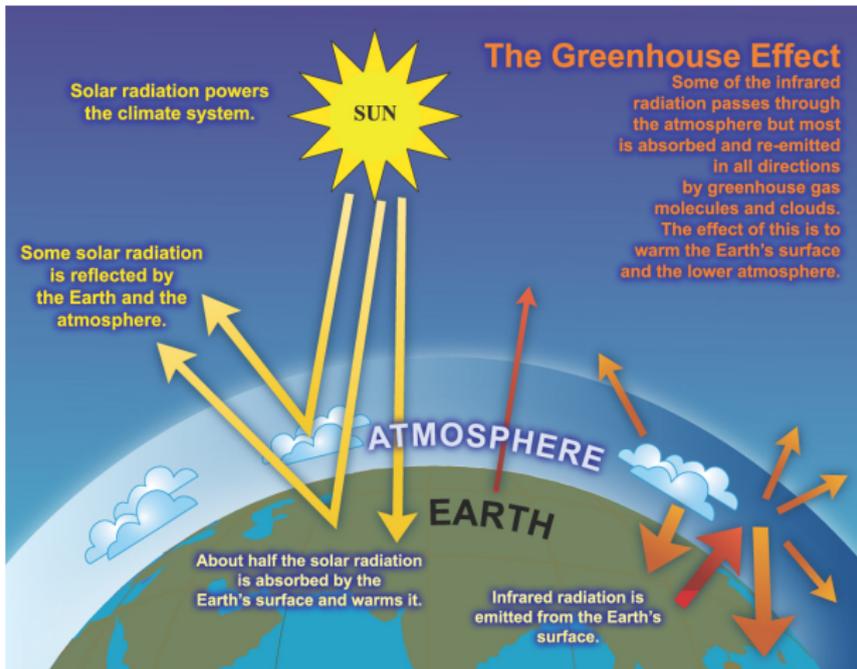
# Clouds



Clouds are a common feature Q:  
What is the global mean annual  
cloud cover?

They have a major impact on both  
the hydrological cycle and radiation  
budget (refer to earlier processes  
schematics)

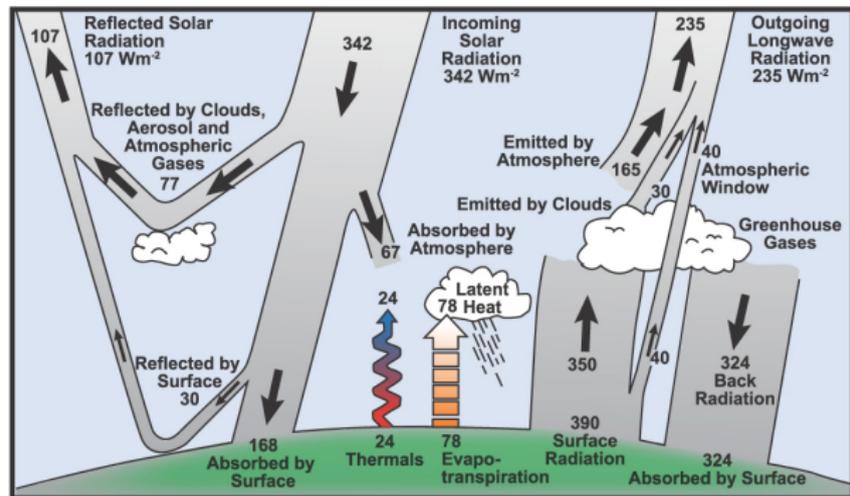
# Atmospheric Processes



**Q: What would be the global mean annual surface temperature**

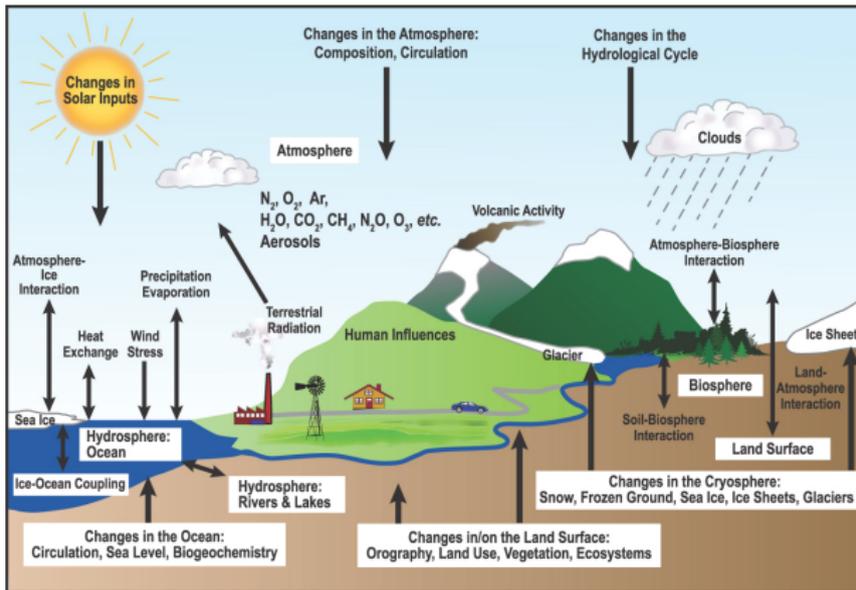
- today?
- in the absence of the atmosphere?
- with an atmosphere but without clouds or aerosols?

# Atmospheric Energy partition



This figure gives more details of the balance, showing how the radiative budget is affected by clouds for example. Note also that the earth's surface loses energy via latent and sensible heat fluxes in addition to infra-red emission. These warm the atmosphere above which itself emits long-wave radiation to space.

# Atmospheric Processes



The climate system is in fact complicated, depending on a huge array of interacting processes. This course will only tackle the basic aspects of cloud microphysics, radiative interaction with the earth-atmosphere system.

## Ideal Gas

The ideal gas model is a model of matter in which the molecules are treated as non-interacting point particles which are engaged in a random motion that obeys conservation of energy.

The model tends to fail at lower temperatures or higher pressures, when the molecules come close enough that they start interacting with each other, and not just with their surroundings. This is usually associated with a phase transition.

The **equation of state for an ideal gas** relates its pressure  $p$ , volume  $V$  and temperature  $T$  by

$$pV = NkT = \nu R^* T, \quad (1)$$

where

- $N$  is the number of identical molecules
- $\nu$  is the number of moles of gas
- $k = 1.3806 \times 10^{-23} \text{ J K}^{-1}$  is Boltzmann's constant,
- $R^* \equiv N_a k = 8.341 \text{ J mol}^{-1} \text{ K}^{-1}$  is the universal gas constant,
- with  $N_a = 6.022 \times 10^{23} \text{ mol}^{-1}$  Avogadro's number.

As pointed out in the introduction, the Earth's atmosphere is a mixture of gases, mostly nitrogen, oxygen and argon, trace gases like carbon dioxide, ozone and methane, and variable amounts of water in its three physical phases (see Table 1).

Neglecting water vapour for the moment, the dry atmosphere can be taken to a very good approximation as an ideal gas.

In general, it is useful to formulate the thermodynamic relations in terms of *intensive* variables. An intensive variable is one whose value does not depend on the amount of matter in the system, like the temperature or pressure. In contrast, an *extensive* variable, depends on the size of the system. For instance, the internal energy of a gas is an extensive variable since if we double its size, all else being kept equal, its internal energy will double.

Given a system whose volume  $V$  contains an amount of mass  $M$ ,

$$v = \frac{V}{M} \quad (2)$$

denotes the *specific volume* of the system. In principle, every extensive variable can be converted to its corresponding intensive form by normalizing it by the amount of matter it describes. We will (generally, but not always!) use lower case letters to denote specific intensive quantities, as opposed to their non-intensive counterpart for which we will use capital letters, e.g. the specific volume  $v$  and the volume  $V$ .

We would like to write the gas law in terms of kilograms rather than moles and use the molecular weight of a species  $i$ ,  $m_i$  (g/mol), which is the mass of 1 mole of identical molecules in grams.

If the total mass of the gas is  $M_i$ , we can express the gas law as,

$$pV = \frac{M_i}{m_i} R^* T. \quad (3)$$

The molecular weight can be substituted out by using the *specific gas constant*  $R_i$  for a species  $i$ , which is defined as

$$R_i \equiv R^* / m_i. \quad (4)$$

## Partial Pressure

For a mixture of ideal gases the partial pressure  $p_i$  of the  $i^{\text{th}}$  gas is defined as the pressure  $p_i$  that it would have if the same mass ( $M_i$ ) existed alone at the same temperature  $T$  and occupying the same volume  $V$ .

By (3) and (4) the partial pressure for an ideal gas can thus be expressed as

$$p_i = \frac{T}{V} M_i R_i. \quad (5)$$

## Dalton's Law

According to **Dalton's law of partial pressures**, the total pressure  $p$  of a mixture of (ideal) gases is the sum of the pressures  $\sum p_i$  of each species  $i$  as if it alone occupied a volume  $V$ ,

$$p = \frac{T}{V} \sum_i (M_i R_i) = \rho R_m T. \quad (6)$$

If  $M_{tot} = \sum_i M_i$  is the total mass, then  $R_m = 1/M_{tot} \sum_i M_i R_i$  the specific gas constant for the mixture, and  $\rho = M_{tot}/V$  is the density.

Equation. (6) is the form of the gas law generally used in meteorology.

Table 1 shows the value of the gas constant for dry air,  
 $R_d = 287.05 \text{ J kg}^{-1} \text{ K}^{-1}$ .

Gas	Molecular weight $m_i \text{ [g mol}^{-1}\text{]}$	Molar fraction	Mass fraction $M_i/M_{tot}$	Specific gas constant $R_i \text{ [J kg}^{-1} \text{ K}^{-1}\text{]}$	$(M_i/M_{tot})R_i$ $\text{[J kg}^{-1} \text{ K}^{-1}\text{]}$
NO <sub>2</sub>	28.013	0.7809	0.7552	296.80	224.15
O <sub>2</sub>	31.999	0.2095	0.2315	259.83	60.15
Ar	39.948	0.0093	0.0128	208.13	2.66
CO <sub>2</sub>	44.010	0.0003	0.0005	188.92	0.09
total		1.0000	1.0000		287.05

**Table:** Main components of dry atmospheric air (source Smithsonian Meteorological Tables).

## Reminders

- 1 What is the definition of a Ideal Gas?
- 2 Can dry air be considered an ideal gas?
- 3 What is Dalton's law?

## first law of thermodynamics

The **first law of thermodynamics** is a statement of two empirical facts:

- 1 Heat is a form of energy
- 2 Energy is conserved

Thus, the internal energy of a closed system ( $U$ ) can change only if heat ( $Q$ ) is added or if work ( $W$ ) is done **on**<sup>1</sup> the system by its surroundings:

$$dU = dQ + dW \quad (7)$$

and for a unit mass of gas:

$$du = dq + dw \quad (8)$$

---

<sup>1</sup>Attention! Sometimes defined as work done **by** the gas thus changing the sign convention for  $dW$

The rate of work is given by

$$dW = -pdV, \quad (9)$$

such that (7) can be expressed as

$$dU = dQ - pdV. \quad (10)$$

If the equation of state is governed by three variables ( $p, v, T$ ) then we can write

$$p = f_1(v, T) \quad , \quad v = f_2(p, T) \quad \text{or} \quad T = f_3(p, v). \quad (11)$$

In other words, there are only two independent state variables,

$$dp = \frac{\partial f_1}{\partial v} dv + \frac{\partial f_1}{\partial T} dT \quad , \quad dv = \frac{\partial f_2}{\partial p} dp + \frac{\partial f_2}{\partial T} dT \quad , \quad dT = \frac{\partial f_3}{\partial p} dp + \frac{\partial f_3}{\partial v} dv. \quad (12)$$

If we differentiate state variables other than  $p, v,$  or  $T,$  for instance, the specific internal energy  $u,$  we must specify which set of thermodynamics parameters we use. Otherwise  $(\partial u / \partial p)$  is ambiguous and depends on the choice of thermodynamic coordinates, for example whether  $u$  is defined as a function of  $p$  and  $v$  or as a function of  $p$  and  $T.$

## Notation for differentiating

To express the rate of change of  $u$  as a result of an isothermal process, i.e. a change in the pressure  $p$ , we write

$$\frac{\partial u(p, T)}{\partial p} \equiv \left( \frac{\partial u}{\partial p} \right)_T \quad (13)$$

meaning that  $p$  and  $T$  are chosen as thermodynamical coordinates and that the the temperature is held constant for this process.

A measure of the quantity of heat needed to raise the temperature of one gram of a substance by 1 °C is called the heat capacity  $C$ , and is defined as:

$$C = dQ/dT. \quad (14)$$

But this definition is incomplete. There are many ways to add heat to a system. One could add heat to a system at *constant volume* or at *constant pressure*, or one could add heat as both the volume and pressure change.

Q: One can even add heat to a system even at constant temperature - how?

We now define the specific heat capacity at constant pressure,  $c_p$  and the specific heat capacity at constant volume,  $c_v$ . These are very different.

$$c_v = \left( \frac{\partial q}{\partial T} \right)_v. \quad (15)$$

$$c_p = \left( \frac{\partial q}{\partial T} \right)_p. \quad (16)$$

For dry air,  $c_p = 1005 \text{ J kg}^{-1} \text{ K}^{-1}$  and  $c_v = 718 \text{ J kg}^{-1} \text{ K}^{-1}$

If one chooses the specific volume  $v$  and the temperature  $T$  as thermodynamic coordinates, then the specific internal energy  $u$  can be expressed as,

$$du = \left( \frac{\partial u}{\partial v} \right)_T dv + \left( \frac{\partial u}{\partial T} \right)_v dT = \left( \frac{\partial u}{\partial T} \right)_v dT. \quad (17)$$

where the last equality follows from the fact that the internal energy of an ideal gas does not depend on its volume,

$$\left( \frac{\partial u}{\partial v} \right)_T = 0. \quad (18)$$

(18) implicitly assumes that the intermolecular forces are negligibly small, and therefore is applicable only to an ideal gas.

The density in the gas law (6) can be replaced by the specific volume, giving  $p\nu = RT$ . Substituting this form into (10) and using (17) gives

$$dq = \left( \frac{\partial u}{\partial T} \right)_v dT + p d\nu, \quad (19)$$

where  $dq$  indicates the differential amount of heat added. For an *isometric* process  $d\nu = 0$ , and thus from our definition of  $c_v$ :

$$dq = c_v dT + p d\nu, \quad (20)$$

Eqn. 20 is still not in a form useful in meteorology due to the difficulty of measuring volume of an air mass. We therefore use the first law again to write

$$dq = c_v dT + R dT - v dp, \quad (21)$$

$$dq = (c_v + R) dT - v dp, \quad (22)$$

For an isobaric process  $dp = 0$ , it is clear that

$$c_p = c_v + R. \quad (23)$$

which leads to the form of the first law commonly used in meteorology since it is formed in measurable quantities:

$$dq = c_p dT - v dp, \quad (24)$$

A parcel of air can be subject to input of heat and changes in pressure. **Q: Name some common atmospheric diabatic processes**  
First we shall consider the case where no heat is input.

### Adiabatic Process

An **adiabatic process** is one where there is no heat exchange,  $dq = 0$ , thus

$$c_p dT = v dp, \quad (25)$$

Adiabatic processes have special significance as many atmospheric motions can be approximated as adiabatic. Assuming that the atmosphere is in a hydrostatic balance<sup>2</sup>,  $dp/dz = -\rho g$ , then (25) can be written

$$c_p dT = -v \rho g dz = -g dz, \quad (26)$$

---

<sup>2</sup>you have already come across the notion of hydrostatic balance in an earlier course, but as a reminder hydrostatic equilibrium exists if the force due to gravity is balanced by the vertical pressure gradient force, leading to this relationship. Hydrostatic equilibrium is widely satisfied over horizontal scales exceeding 10km, and thus the assumption is used to simplify the governing equations of numerical models of the atmosphere that use a grid-cell size exceeding this threshold

This gives the adiabatic lapse rate as

$$\frac{dT}{dz} = -\frac{g}{c_p} \quad (27)$$

Thus if we lift a dry air parcel without heat exchange its temperature will fall by by  $9.8 \text{ K km}^{-1}$

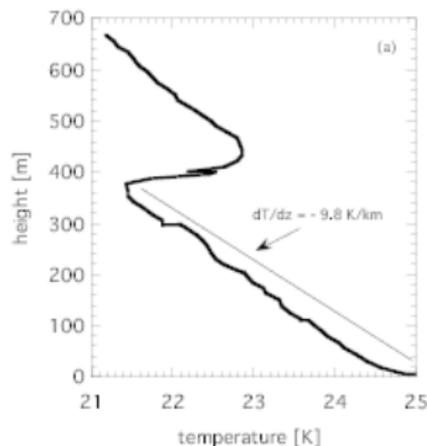


Figure 1 shows the measured lapse rate on one particular day measured in Holland. The lapse rate is seen to be close to adiabatic.

**Q:** Can you think as to why this may be? **Hint:** Stability

**Figure:** Vertical profile of the temperature  $T$  for a dry air parcel on a particular day measured in Holland.

Using the equation of state we get

$$c_p dT = v dp = \frac{RT}{p} dp \quad (28)$$

$$\frac{dT}{T} = \frac{R}{c_p p} dp \quad (29)$$

### Poisson's Equation

Eqn. 29 can be integrated to give **Poisson's Equation**:

$$\frac{T}{T_0} = \left( \frac{p}{p_0} \right)^{\frac{R_d}{c_p}} \quad (30)$$

where  $T_0$  and  $p_0$  are constants of integration. **see Exercise:**  
**Derive this equation.**

Note that we have reintroduced the subscript  $d$  on the gas constant to emphasize that this considers dry air.

## Potential Temperature

If we set the reference pressure  $p_0$  to 1000 hPa then  $T_0$  is defined as the **potential temperature** (for dry air) which is commonly denoted  $\theta$ :

$$\theta = T \left( \frac{p_0}{p} \right)^{\frac{R_d}{c_p}}. \quad (31)$$

*The potential temperature  $\theta$  can be interpreted as the temperature a parcel would have if it were displaced adiabatically to a reference height where the pressure is  $p_0$ , which is usually taken  $p_0 = 1000$  hPa. In other words, a parcel with temperature  $T$  at pressure level  $p$  will have a potential temperature  $\theta$ , which value is equal to the temperature  $T_0$  at the pressure level  $p_0$ .*

The **lapse rate** of the potential temperature can be obtained by differentiating (31) with respect to height

$$\frac{d\theta}{dz} = \frac{\theta}{T} \left( \frac{dT}{dz} - \frac{R_d T}{p c_p} \frac{dp}{dz} \right) \quad (32)$$

Assuming that the atmosphere is in a hydrostatic balance,  $dp/dz = -\rho g$ , then with aid of the gas law (32) can be written as

$$\frac{d\theta}{dz} = \frac{\theta}{T} \left( \frac{dT}{dz} + \frac{g}{c_p} \right). \quad (33)$$

Since  $-\frac{g}{c_p} = \Gamma_d$  is the **dry adiabatic lapse rate**, we conclude that if the temperature profile follows the dry adiabatic lapse rate, the potential temperature is constant with height.

A **reversible** process is defined as

$$\oint \frac{\delta Q}{T} = 0. \quad (34)$$

This integral is independent of the path. The specific entropy  $\phi$  is defined as

$$d\phi = \frac{dq}{T}. \quad (35)$$

Unlike heat and work, the entropy is a state variable, thus we can speak of the entropy in state  $A$  or  $B$ . Note that, with the temperature in the denominator of (34), heat and entropy are not linearly related.

An isentropic process (iso = "equal" (Greek); entropy = "disorder") is one during which the entropy of the system remains constant; a reversible, adiabatic process is isentropic.

The second law of thermodynamics states that a process will tend to increase the entropy of a system, thus  $Td\phi = du + pdv \geq 0$ . The relevance of entropy to meteorologists is clear if we define the potential temperature in terms of the entropy:

$$d\phi = c_p d \ln \theta. \quad (36)$$

**Exercise: Derive eqn. 36. Hint: Take logs of eqn. 31 and differentiate**

The heat added in a cyclic process is<sup>3</sup>

$$\oint dq = \oint Td\phi = c_p \oint Td(\ln\theta), \quad (37)$$

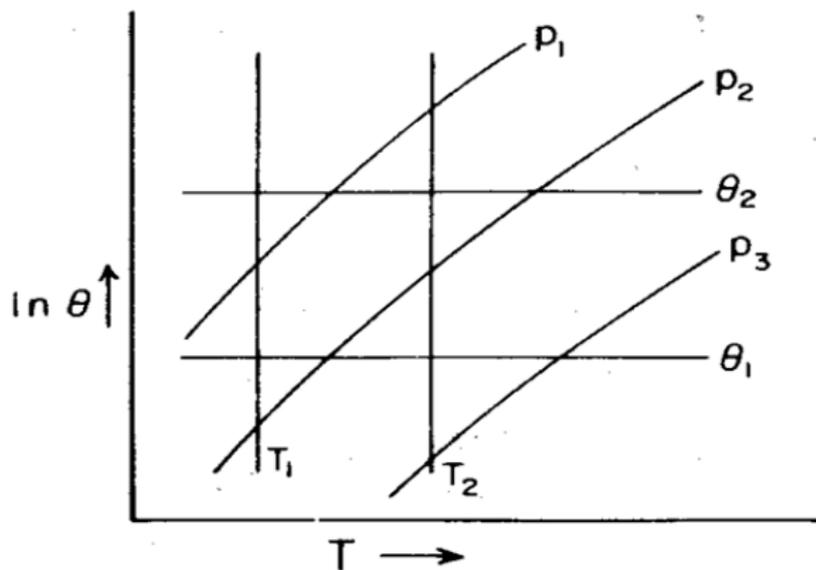
where the last equality follows from using (36). Thus it is clear that a chart that has perpendicular coordinates of temperature  $T$  and entropy  $\phi$ , or equivalently  $T$  versus  $\ln\theta$  will have the properties of **equal areas=equal energy** ; useful properties of a thermodynamic diagram.

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<sup>3</sup>This can also be derived directly from the first law and the hydrostatic relation.

## tephigrams

Such a chart is known as a **tephigram**. Using (31) one can plot  $p$  as a function of the two ordinates. Note that the lines of constant pressure, (isobars), are not exactly straight lines.



**Figure:** The tephigram basic structure

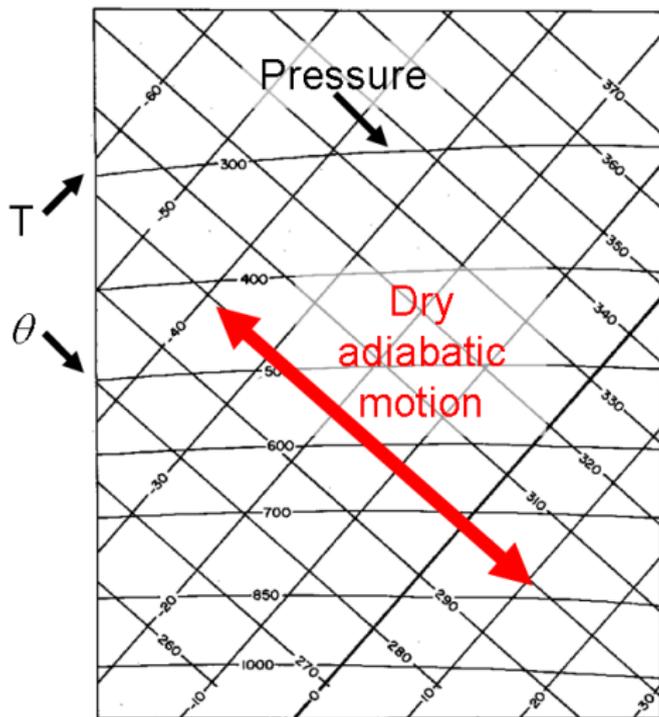


FIG. 1.7. Skeleton of a tephigram. Isobars are approximately horizontal, labeled in mb. Isotherms in deg C go upwards and to the right. Dry adiabats are normal to the isotherms and are labeled according to potential temperature (deg K).

Meteorologists are used to using pressure as a vertical height coordinate and thus rotate the chart to make the curved pressure lines almost horizontal, as shown in Figure 3.

To calculate the buoyancy force on a parcel of air we will assume that the atmosphere is in hydrostatic balance and that pressure field is not perturbed by the presence of the air parcel. The parcel of (dry) air has a with volume  $V$ , temperature  $T$  and density  $\rho$ . It displaces an equal volume of ambient environmental (noted by subscript  $env$ ) air having properties  $T_{env}, \rho_{env}$ . The downward force on the parcel is  $\rho gV$  and on the air it displaces  $\rho_{env}gV$ . The upward force  $-V\frac{\delta p}{\delta z}$  is identical for both. Hence the net upward force is  $Vg(\rho_{env} - \rho)$  and the buoyancy force per unit mass

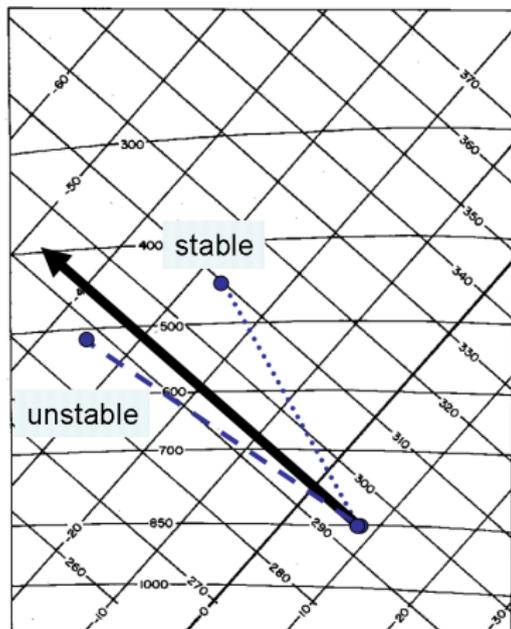
$$F_B = g \left( \frac{\rho_{env} - \rho}{\rho} \right) \quad (38)$$

Now using Poisson's eqn, (38) can be written

$$F_B = g \left( \frac{\theta - \theta_{env}}{\theta_{env}} \right) \quad (39)$$

If a parcel of air is warmer (cooler) than its environment it will be subject to a net upwards (downwards) buoyancy force and will accelerate upwards (downwards), and is usually referred to as **positively (negatively) buoyant**. Equal temperatures are referred to as **neutral buoyancy**.

A parcel of air that is neutrally buoyant and subjected to a small vertical displacement may become positively or negatively buoyant, depending on whether the parcel's temperature changes more or less rapidly than the environmental lapse rate.



The environmental  $\theta$  lapse rate is  $\frac{d\theta_{env}}{dz}$  and we will assume the lifted parcel cool adiabatically (reminder: what does this mean?) and thus  $\theta$  of the parcel is invariant.

We can thus define three situations:

$$\frac{d\theta_{env}}{dz} < 0 : \text{Unstable}$$

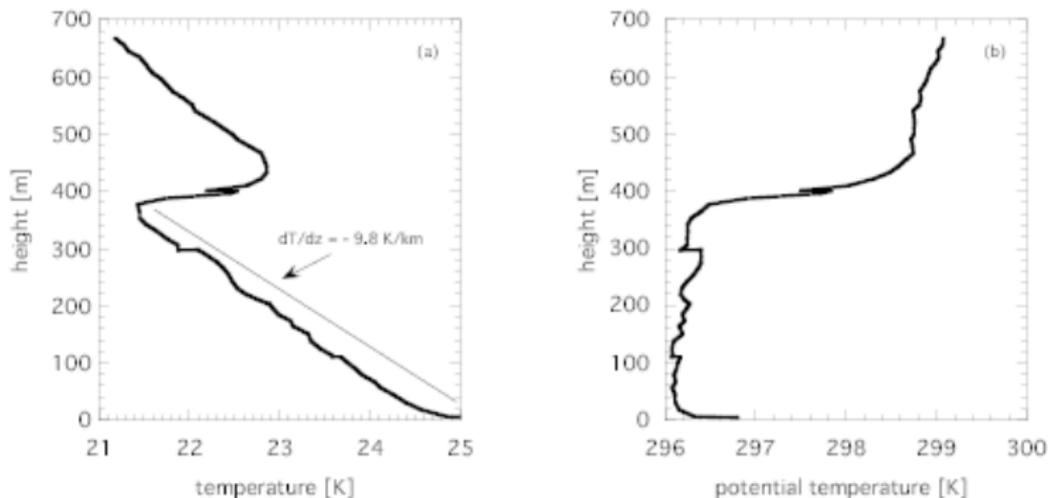
$$\frac{d\theta_{env}}{dz} = 0 : \text{Neutral}$$

$$\frac{d\theta_{env}}{dz} > 0 : \text{Stable}$$

With the example stable and unstable profiles shown in Fig. 4

**Figure:** Two idealized profiles showing (dotted/dashed) stable/unstable

We will now return to the Cabauw profiles (Fig. 1). Q: Why do you think the profile is near neutral?



**Figure:** Vertical profile of the potential temperature  $\theta$  for the Cabauw profile in Fig. 1.



Without instrumentation to measure the profile of temperature one can not say whether a boundary layer is well mixed, however one clue is the presence of a gusty intermittent wind associated with the turbulent eddies, while if the layer is deep enough such that the top of the boundary layer becomes saturated and forms “fair weather cumulus” cloud (see next section) then this is a sure sign that the boundary layer is well mixed and close to neutral stability.

Thus **unstable** layers are quite uncommon except in the lowest few metres of the atmosphere, **neutral** layers are quite common and result from the convective/turbulent reaction to processes that produce instability, and **stable** layers are also common. In this case displaced parcels of air will undergo simple harmonic motion.



Lenticular or lee-wave cloud indicating atmospheric stable layer.

Courtesy UCAR

## Reminders:

- If we express the gas law in terms of mass instead of moles, we switch from using the universal gas constants to a specific gas constant whose value depends on the molecular weight.
- The dry atmosphere and water vapor can be taken to a good approximation as an ideal gas.
- The state of the atmosphere can be expressed by three state variables,  $p$ ,  $v$  and  $T$ , of which two are independent.
- The first law of thermodynamics dictates that energy is conserved.

- 1** Assume that a parcel of air undergoes net diabatic radiative cooling of  $10^{-5} \text{ K s}^{-1}$ . The parcel is also undergoing vertical motion at the same time at a velocity of  $w$ . We will assume that we can neglect the impact of radiation if the heating/cooling associated with the vertical motion is at least ten times greater. What would be the minimum vertical velocity of the parcel for this to be the case? What are typical rates of ascent in cumulus clouds in comparison?
- 2** In the subtropical free (above the boundary layer) troposphere there is an approximate balance between radiative cooling and dry adiabatic descent of the air. Assuming a net radiative cooling of  $10^{-5} \text{ K s}^{-1}$ , what rate must the air descend at to maintain equilibrium?

## Moist Thermodynamics

In the previous section we restricted our analysis to dry air, **we now extend this to include water**, which unlike other atmospheric constituents, can appear in all its three phases: solid, liquid and vapour.

## Equation of state for water vapour

Water vapour can be treated as an ideal gas to a good approximation and thus from the gas law it follows that

$$e = \rho_v R_v T, \quad (40)$$

where  $\rho_v$  is the density of water vapor, and  $R_v$  is the specific gas constant for water vapour which is equal to  $461.5 \text{ J kg}^{-1} \text{ K}^{-1}$ .

In the literature the ratio between the gas constants for dry air and water vapour is used:

$$\epsilon = \frac{R_d}{R_v} = \frac{m_v}{m_d} \quad (41)$$

where  $\epsilon = 0.622$  from the values of the gas constants.

Atmospheric air is a mixture of the ideal gases of dry air and water vapour. From Dalton's law, the total pressure of moist air is the sum of the partial pressures of the dry air and the vapour

$$p = p_d + e. \quad (42)$$

Imagine a closed system consisting of a water body and a vacuum above. The water molecules in the water body are in a state of thermal agitation and the most energetic ones will overcome the inter-molecular attraction and break free of the water body. The rate at which vapor molecules leave the surface depends upon the characteristics of the surface. The process is known as *evaporation*. The vapour vapour pressure increases as a result, which will be denoted  $e$ .

Some of the water molecules will subsequently collide with the water surface and stick to the surface. This process is known as *condensation*, and decreases the vapour pressure. The rate at which vapor molecules arrive at a surface of liquid (cloud drop) or solid (ice crystal) depends upon the *vapor pressure*.

Since the condensation depends on the vapour pressure, it is apparent that in the absence of external perturbations a state of dynamic equilibrium will eventually be reached in which the rates of *condensation* and *evaporation* equal each other (considering only liquid for now). The vapour is said to be **saturated** in this case, and the vapour pressure  $e$  is equal to the so-called **saturation vapour pressure  $e_s$  with respect to water.**

As the evaporation rate is only dependent on temperature, and the condensation rate on the vapour pressure, there are two important consequences to note concerning  $e_s$ :

- 1  $e_s$  depends on  $T$  and  $T$  **only**.
- 2 Saturation is **independent** of the pressure of other gases

## Supersaturation

If the saturation vapour pressure exceed the saturation value  $e > e_s$  then the air is said to be **supersaturated** with respect to liquid water. We shall see that greatly supersaturated states *with respect to liquid water* in the atmosphere are not observed, as the excess water vapour quickly condenses to form liquid cloud droplets. The phrase *liquid water* is emphasized as the situation is not the same when it comes to cloud ice crystals as we shall see.

We now derive the dependence of  $e_s$  on  $T$ . Heat must be supplied to change a unit mass of liquid to vapour at a constant temperature; this is known as the **Latent Heat of Vaporization**, denoted  $L_v$ . Throughout the process of evaporation the  $e_s$  remains constant as it is a function of  $T$  only. For the transition from liquid phase  $q_1$  to vapour phase  $q_2$ ,

$$L_v = \int_{q_1}^{q_2} dq = \int_{u_1}^{u_2} du + \int_{v_1}^{v_2} pdv = u_2 - u_1 + e_s(v_2 - v_1). \quad (43)$$

However, since temperature is also constant we can write

$$L_v = T \int_{q_1}^{q_2} \frac{dq}{T} = T(\phi_2 - \phi_1) \quad (44)$$

Equating (43) and (44) shows

$$u_1 + e_s v_1 - T \phi_1 = u_2 + e_s v_2 - T \phi_2. \quad (45)$$

## Gibbs Function

This implies that the so-called **Gibbs Function**

$$G = u + e_s v - T \phi \quad (46)$$

of a system is constant during isothermal, isobaric changes of phase.

The Gibbs function does vary however as a function of temperature and pressure, and differentiating (46) gives:

$$dG = du + vde_s + e_sdv - Td\phi - \phi dT. \quad (47)$$

But  $Td\phi = dq = du + e_sdv$  giving

$$dG = vde_s - \phi dT. \quad (48)$$

The Gibbs function is independent of phase, such that  $dG_1 = dG_2$ :

$$v_1de_s - \phi_1dT = v_2de_s - \phi_2dT. \quad (49)$$

## Clausius Clapeyron Equation

We can now rearrange (49) to derive the **Clausius Clapeyron Equation**:

$$\frac{de_s}{dT} = \frac{\phi_2 - \phi_1}{v_2 - v_1} = \frac{L_v}{T(v_2 - v_1)}, \quad (50)$$

where the last relationship uses (44). Under ordinary conditions the specific volume for water vapour is much greater than that of liquid,  $v_2 \gg v_1$  allowing the Clausius Clapeyron equation to be approximated as

$$\frac{de_s}{dT} \approx \frac{L_v}{Tv_2} = \frac{L_v e_s}{R_v T^2}, \quad (51)$$

where the final relationship uses the Gas Law once again.

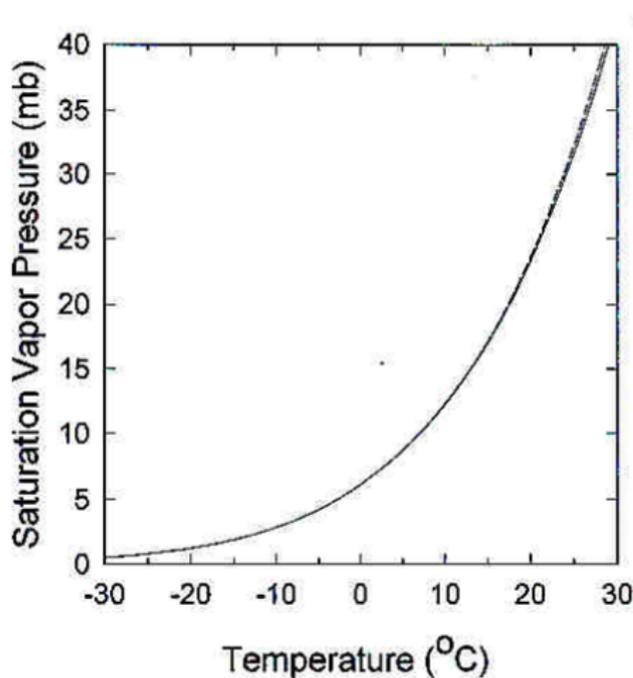
At first sight it would appear to be straightforward to integrate (51) to give  $e_s$  as a function of  $T$ , but this is complicated by the fact that the Latent heat of vaporization  $L_v$  is itself a function of temperature. Fortunately this dependence is weak, with  $L_v$  varying by 6% from  $-30^\circ\text{C}$  to  $+30^\circ\text{C}$ . Neglecting this dependence as a first approximation, integrating (51) gives

$$e_s = e_{s0} \exp \left[ \frac{L_v}{R_v} \left( \frac{1}{T_0} - \frac{1}{T} \right) \right]. \quad (52)$$

$e_{s0}$  is the vapour pressure at temperature  $T_0$ , and at  $T = 0^\circ\text{C}$  we have  $e_{s0} = 6.11 \text{ hPa}$ , and a value of  $2.50 \times 10^6 \text{ J kg}^{-1}$  can be assumed for  $L_v$ . Substituting these values gives an approximate for  $e_s$  as

$$e_s(T) = A e^{\frac{-B}{T}} \quad (53)$$

where the constants are  $A = 2.53 \times 10^8 \text{ kPa}$  and  $B = 5.42 \times 10^3 \text{ K}$ .



$e_s$  doubles for every  $10^\circ\text{C}$  increase in temperature. Note that the nonlinearity of the saturation vapour pressure has important consequences for mixing of air parcels as is thus relevant for processes such as atmospheric convection.

**Figure:** The saturation vapour pressure as a function of temperature

Below the freezing point at  $T = 0^\circ\text{C}$  the Clausius Clapeyron equation describes the saturation pressure of **supercooled** liquid water (liquid water at temperatures below  $0^\circ\text{C}$ ), and this is still relevant as we will see later that ice crystals do not form at  $T = 0^\circ\text{C}$  and supercooled liquid cloud droplets are common. Of course at temperatures below freezing ice crystals may be present, and the saturation vapour pressure of ice, denoted  $e_i$  is also described by (51) but with  $L_v$  replaced by  $L_s$ , which is the **Latent heat of Sublimation**, for which a value of  $2.83 \times 10^6 \text{ J kg}^{-1}$  can be used. The ratio between the two at subfreezing temperatures is

$$\frac{e_s(T)}{e_i(T)} = \exp \left[ \frac{L_f}{R_v T_0} \left( \frac{T_0}{T} - 1 \right) \right], \quad (54)$$

where  $L_f$  is the **Latent Heat of Fusion** and is equal to  $L_s - L_v$ .

## The dependency of the latent heat of vaporization on the temperature

To calculate the dependency of  $L_v$  on  $T$  we use (43) and note again that  $v_2 \gg v_1$  and that  $e_s v_2 = R_v T$ . Then differentiating we get

$$\frac{dL_v}{dT} = \frac{du_2}{dT} - \frac{du_2}{dT} + R_v \quad (55)$$

Now  $\frac{du_2}{dT} = c_{vv}$  which is the **specific heat capacity of water vapour at constant volume** and  $\frac{du_1}{dT} = c_{vl}$  is the **specific heat capacity of liquid water at constant volume**.

We again note the relationship that the **specific heat of water vapour at constant pressure**  $c_{pV} = c_{vV} + R_V$ , and then integrate to get

$$L_v(T) = L_0 - (c_{vI} - c_{pV})(T - T_0), \quad (56)$$

where  $L_0 = L(T_0)$  is the constant of integration. Both  $c_{vI}$  and  $c_{pV}$  vary as a function of temperature and pressure, but this variation is weak amounting to less than 3% over the full range of tropospheric conditions, and the constant values of  $c_{pV} = 1870 \text{ J kg}^{-1} \text{ K}^{-1}$  and  $c_{vI} = 4187 \text{ J kg}^{-1} \text{ K}^{-1}$  will be assumed here, while  $c_{vV} \simeq 1410 \text{ J kg}^{-1} \text{ K}^{-1}$ . For a complete list of gas constant and specific heat values refer to Appendix 2 of Emanuel (1994).

In addition to the **vapour pressure**  $e$  and the **vapour density**  $ho_v$ , there are alternative ways to describe the water vapour content of air that are more commonly used in Meteorology.

## Mixing Ratio $r_v$

This is defined as the mass of water vapour per unit mass of **dry** air

$$r_v = \frac{M_v}{M_d} = \frac{\rho_v}{\rho_d}. \quad (57)$$

From the equation of state,  $\rho_d = \frac{p-e}{R_d T}$ , so that

$$r_v = \frac{\epsilon e}{p - e} \quad (58)$$

The saturation mixing ratio  $r_s$ , *with respect to liquid water*, is defined by replacing  $e$  with  $e_s$ , and is a function of both pressure and temperature.

## Specific Humidity $q_v$

This is defined as the mass of water vapour per unit mass of **moist** air

$$q_v = \frac{\rho_v}{\rho} = \frac{\rho_v}{\rho_d + \rho_v}, \quad (59)$$

and using the same substitution as above

$$q_v = \frac{\epsilon e}{p - (1 - \epsilon)e} \quad (60)$$

The saturation specific humidity is defined by replacing  $e$  with  $e_s$ .

At all normal atmospheric conditions  $e \ll p$ , implying that in practice

$$q_v \approx r_v \approx \frac{\epsilon e}{p}, \quad (61)$$

The difference between  $r_v$  and  $q_v$  is greatest at the surface in the tropics and roughly 2% there<sup>4</sup>.

---

<sup>4</sup>This similarity leads to the two terms being used almost interchangeably in meteorology, not good practice, and one that is not helped by the fact that the notation for the specific humidity and mixing ratio are not standard, with  $q_v$  often used for mixing ratio in the literature. Note also that the subscript  $v$  is not always used.

## Relative Humidity $RH$

The ratio of the vapour pressure to its saturated value:

$$RH = \frac{e}{e_s}. \quad (62)$$

Note that the relative humidity can be approximated as

$$RH \approx \frac{r}{r_s}, \quad (63)$$

however often this is used as an exact relationship in the literature. Some texts such as Rogers and Yau (1989) break this convention altogether and define  $RH$  in terms of mixing ratio and state (62) as an approximate relationship

## Virtual Temperature $T_v$

A *hypothetical temperature*, the virtual temperature is the temperature a sample of dry air would have to have the same density as the same of moist air in question, at the same pressure.

To derive  $T_v$  we recall:

$$p = p_d + e = \rho_d R_d T + \rho_v R_v T = \frac{R^* T}{V} \left( \frac{M_d}{m_d} + \frac{M_v}{m_v} \right), \quad (64)$$

and using the gas law we get

$$p = \frac{\rho R^* T}{(M_d + M_v)} \frac{M_d}{m_d} \left( 1 + \frac{M_v}{M_d} \frac{m_d}{m_v} \right). \quad (65)$$

Recalling the definitions of  $\epsilon = m_v/m_d$  and  $r_v = M_v/M_d$ , (65) can be simplified to ( **Exercise: Show** )

$$p = \rho R_d T \left( \frac{1 + \frac{r}{\epsilon}}{1 + r} \right) \quad (66)$$

Thus the equation of state for moist air can be written as the equation of state for dry air, but using the adjustment factor in brackets. We thus define the virtual temperature as

$$T_v \equiv T \left( \frac{1 + \frac{r_v}{\epsilon}}{1 + r_v} \right) \quad (67)$$

Allowing us to write the equation of state for moist air as

$$p = \rho R_d T_v. \quad (68)$$

Since  $r_v \ll 1$ , by ignoring second order terms the definition of  $T_v$  can be simplified to **Exercise: show**

$$T_v = T \left( 1 + \frac{1 - \epsilon}{\epsilon} r_v \right) \quad (69)$$

We can see that for moist air  $T_v > T$  always since a unit volume of moist air is less dense than a unit volume of dry air. Thus for moist air it is  $T_v$  that should be used in the definition of buoyancy to account for this effect.

We define the *virtual* potential temperature in terms of  $T_v$

$$\theta_v \equiv T_v \left( \frac{p_0}{p} \right)^{\frac{R_d}{c_p}}. \quad (70)$$

and this quantity is conserved in adiabatic motion of moist air.

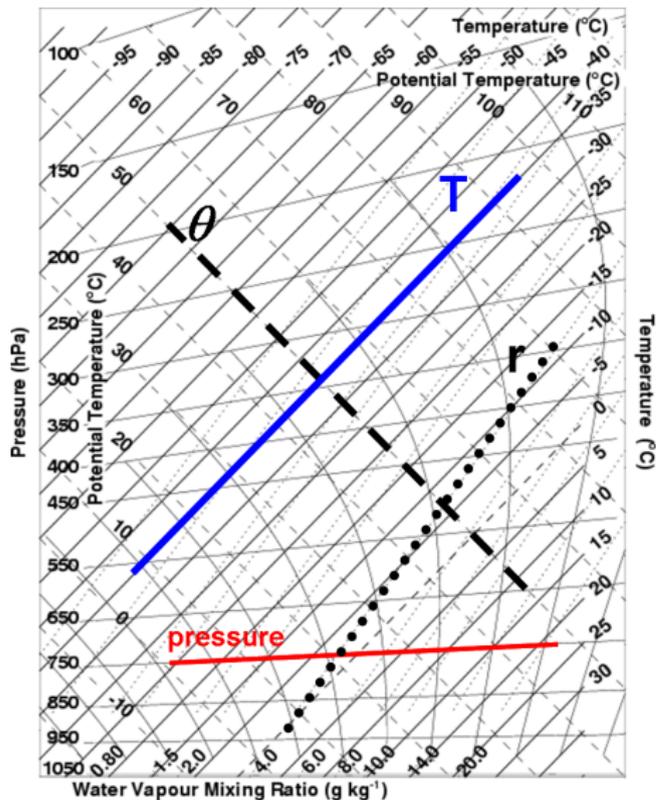
We can apply the equation of state to moist air by replacing  $T$  with  $T_v$ , or we can alternatively use the gas constant for moist air  $R_m$ :

$$p v = R_d T_v = R_m T \quad (71)$$

Thus we can see that  $R_m$  is related to  $R_d$  by

$$R_m = R_d \left( \frac{1 + \frac{r_v}{\epsilon}}{1 + r_v} \right). \quad (72)$$

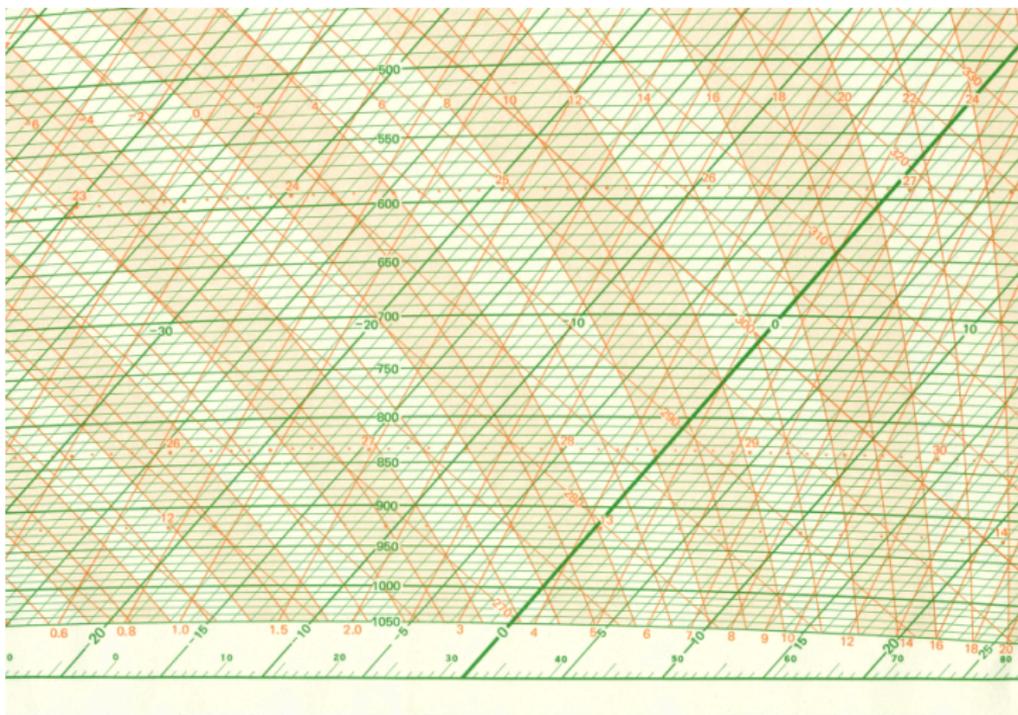
Take care not to confuse  $R_m$  with  $R_v$ .



**Figure:** A blank tephigram with

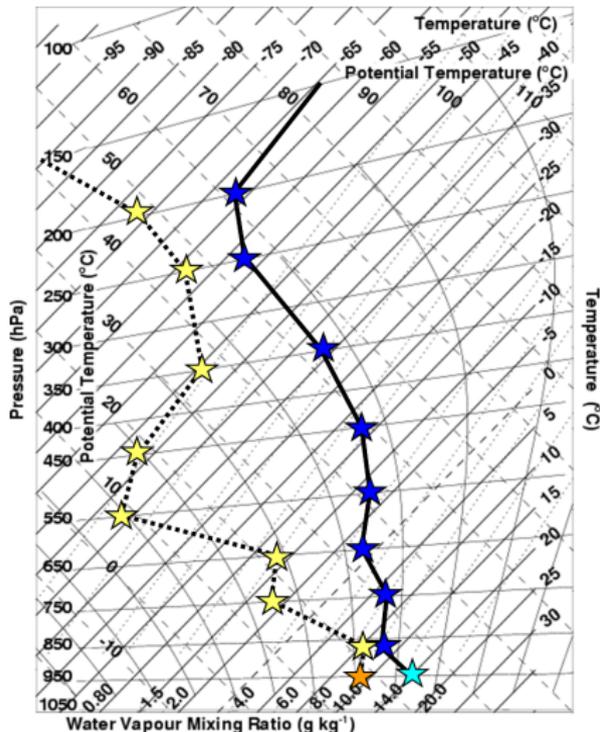
## Water Vapour in Tephigrams

We now return to the tephigram introduced in the previous section and add lines of constant (isopleths of) saturation mixing ratio to the tephigram, recalling from (58) that  $r_s$  is a function of pressure and temperature (Fig.7). **Q:** This tephigram has an error, such that it is *not* a true thermodynamic chart. What is it?



As an aside we reproduce a scanned correct tephigram in Fig. 8, as used and produced by the UK Meteorological Office, unfortunately only for the lower half of the troposphere to 500 hPa.

**Figure:** A correct tephigram. Courtesy EWG at McGill.



**Figure:** tephigram with  $T$  and  $r$  soundings plotted

Thus if at a pressure of 950 hPa we measure a temperature of  $20^{\circ}\text{C}$ , we can plot this on the tephigram, marked as a cyan star in Fig. (9). Likewise if we measure a mixing ratio of  $r = 10\text{gkg}^{-1}$  we can also plot this using the mixing ratio isopleths, marked as an orange star. Making measurements through out the atmosphere (e.g. by a balloon) we can plot a sounding of temperature and humidity, as show by the yellow and blue stars.)

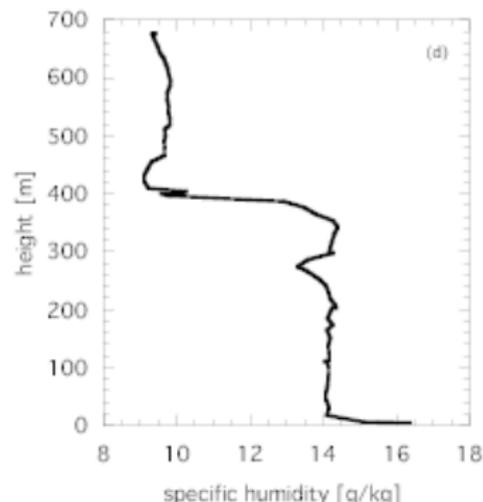
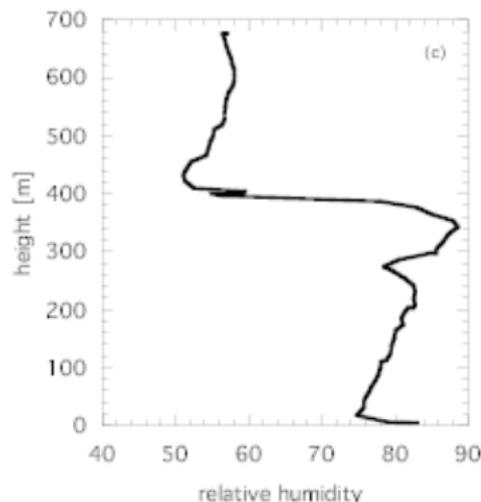
**Tephigram Ex 1** Using the correct tephigram shown in Fig. (8) plot the sounding data given in table 2.

$p(\text{hPa})$	$T$ ( $^{\circ}\text{C}$ )	$r$ ( $\text{g kg}^{-1}$ )
1000	10.0	5.0
950	6.5	4.7
900	3.1	4.5
850	2.5	3.2
800	0.0	2.8
700	-5.1	1.4
600	-12.0	1.3
500	-17.6	0.9

**Table:** An arbitrary atmospheric sounding

- 1 How deep is the planetary boundary layer (PBL) in hPa?
- 2 Is the PBL neutral, stable or unstable ?
- 3 What is the  $RH$  at 1000 hPa and 900 hPa?
- 4 What is  $T_d$  at 1000 hPa?
- 5 Is the PBL “well mixed” in humidity?
- 6 What is the potential temperature  $\theta$  of an air parcel at  $p=800\text{hPa}$ ?
- 7 If this air parcel undergoes adiabatic ascent until it reaches a pressures of  $p=700\text{hPa}$ . What is its new  $T$  and  $\theta$ ?

We will now return to the Cabauw profiles (Fig. 1) and examine the humidity measurements taken during the balloon ascent, which are shown in Fig. 10. These show the sharp drop in humidity at the boundary layer top at 400m and that the humidity is roughly constant through the boundary layer. From the  $RH$  profile, we can see relative humidity increasing through the boundary layer and that the top of the boundary layer is almost saturated.



## Tephigram Ex 2

Assuming that  $r_v \approx q_v$  plot the tephigram of the Cabauw balloon ascent reading  $T$  (or  $\theta$ ) and  $q$  values at the levels of 50m, 200m, 260m, 300m, 350m, 400m, 450m and 650m from Figs. 1 and 10. To convert height into a pressure coordinate simply assume that  $p = 1000$  hPa at the surface and that pressure drops with a scale-height of 7km, i.e.  $p = p_0 \exp(-z/7000)$ . You will see how the profiles of  $T$  and  $q$  in the boundary layer lie close to their respective isopleths of conserved quantities. What is happening at  $z = 260m$  do you think?

Analogous to the water vapour variables described above we can define similar variables that describe the quantity of liquid water or ice in air

- Absolute liquid (ice) water ice density<sup>5</sup> :  $\rho_l$  or  $L$   $\text{kg m}^{-3}$  (or  $\rho_i$ )
- Liquid (ice) water mixing ratio:  $r_l = \frac{\rho_l}{\rho_d}$   $\text{kg kg}^{-1}$  (or  $r_i$ )
- Specific liquid (ice) water content:  $q_l = \frac{\rho_l}{\rho}$   $\text{kg kg}^{-1}$  (or  $q_i$ )

The total water mixing ratio is the sum of the three phases

$$r_t = r_v + r_l + r_i.$$

---

<sup>5</sup>Attention! The absolute liquid water density here is the total mass of liquid water droplets divided by a unit volume of cloudy air - this is not to be confused with the density of liquid water which will denoted  $\rho_L$ . To avoid confusion, the rest of this course always uses liquid water mixing ratio or specific water content, or we will use the notation  $L$  instead of  $\rho_l$

Additionally, analogous to the  $T_v$  we can define the density temperature<sup>6</sup>  $T_\rho$  which is the temperature dry air would have to have equal density to moist cloud air

$$T_\rho \equiv T \left( \frac{1 + \frac{r_v}{\epsilon}}{1 + r_t} \right) \quad (73)$$

Similarly to  $T_v$ , if we assume  $r_l \ll 1$  and  $r_i \ll 1$  and ignore second order terms we get

$$T_\rho \simeq T \left( 1 + \frac{1 - \epsilon}{\epsilon} r_v - r_l - r_i \right) \quad (74)$$

The last two terms show how liquid and ice make a parcel more dense, and this effect is referred to as the **water loading effect**.

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<sup>6</sup>Again, there is not agreement in the literature concerning the virtual temperature  $T_v$  which is sometimes defined to include liquid and ice. Here we follow the notation of Emanuel (1994)

### Specific heat of moist air $c_{vm}$ and $c_{pm}$

The presence of moisture alters the specific heat capacity of air. We add heat to a sample of air consisting of a unit mass (1kg) of dry air and  $r$  kilograms of water vapour

$$(1 + r)dq = c_v dT + r c_{vV} dT \quad (75)$$

this gives

$$c_{vm} = \frac{dq}{dT} = c_v \left( \frac{1 + \frac{c_{vV}}{c_v} r}{1 + r} \right) \quad (76)$$

As  $c_{vv}/c_v = 1.96 \approx 2$  then ignoring second order terms,

$$c_{vm} \approx c_v(1 + r) \quad (77)$$

Likewise the **specific heat at constant pressure** can be approximated

$$c_{pm} \approx c_p(1 + 0.9r) \quad (78)$$

As  $r < 10^{-2}$  these correction factors can generally be neglected.

## Reminders:

- What does it mean for air to be saturated?
- What is  $e_s$  a function of?
- What was one of the problems to get an expression  $e_s = F(T)$ ?
- Q: Can you think of some processes by which air can become saturated?



**Figure:** We will now start to consider processes relevant to deep moist convection so here is a superfluous photo to motivate you (source unknown)

## Ways of reaching saturation

There are several processes by which a parcel of air may become saturated which are all relevant to cloud physics:

- Diabatic cooling (e.g. radiation)
- Adiabatic cooling (e.g. ascent)
- Evaporation (e.g. of precipitation falling through parcel)

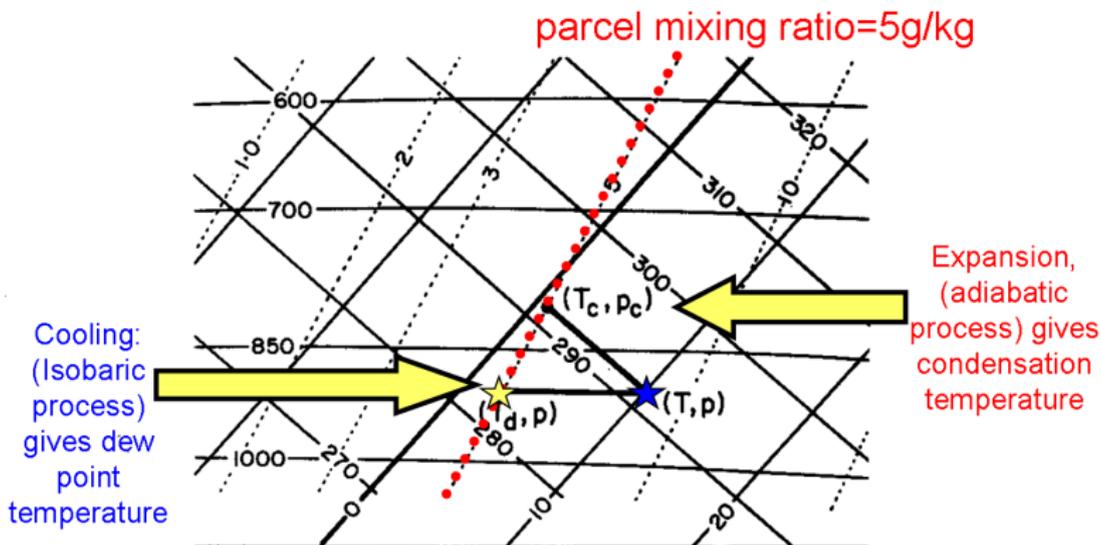


FIG. 2.2. Temperature, dew point, and isentropic condensation temperature, indicated on a tephigram. In the example shown the sample of air at  $10^{\circ}\text{C}$ ,  $900\text{ mb}$ , is assumed to have a mixing ratio of  $5\text{ g/kg}$ . Its dew point, found from the intersection of the  $900\text{ mb}$  isobar and the  $5\text{ g/kg}$  vapor line, is  $2.2^{\circ}\text{C}$ . Its isentropic condensation point, found from the intersection of the adiabat through  $(T, p)$  with the  $5\text{ g/kg}$  vapor line, is at  $0.7^{\circ}\text{C}$  and approximately  $800\text{ mb}$ .

## Diabatic Cooling: Dew Point Temperature $T_d$

As air is cooled isobarically,  $r$  is conserved, and the air will reach saturation when  $T$  is such that

$$r_s(T) = r. \quad (79)$$

This temperature is known as the **Dew Point Temperature  $T_d$** .  
One can write equivalently  $e(T) = e_s(T_d)$ .

## Adiabatic Cooling: Condensation Temperature $T_c$

As air is cooled adiabatically,  $\theta_v$  is conserved, and the air will reach saturation at the **isentropic condensation temperature and pressure**. This pressure level is sometimes referred to as the **lifting condensation level** or LCL.

If ascent and expansion continues condensation will occur (we will see why air does not become supersaturated with respect to liquid later), *thus the temperature will decrease at a slower rate.*

Now in this conceptual model of a parcel of air undergoing ascent in a cumulus cloud, we need to decide what happens to the condensed water: Does it remain in the parcel or will it fall out? **Q:**  
**If the droplets remain in the parcel what do we need to consider?**

**Figure:**

Schematic of parcel ascending and reaching saturation, with condensed water falling out instantaneously

## Pseudo-adiabatic process

If it is assumed that the condensed water remains in the parcel of air then we would need to account for its **water loading** effect and its modification of the **heat capacity**. Moreover when the freezing point is reached we need to consider if and how some of the liquid droplets will freeze, invoking ice processes which are complex, as we shall see. These are issues concerning **microphysics** and **cloud dynamics**, thus involving a **cloud model**.

For now we take the simplest case and assume that all condensate is immediately lost as precipitation, known as the **pseudo-adiabatic** assumption (Fig. 13).

For moist saturated ascent and neglecting correction factor for the specific heat (so we can use  $c_p$ ),

$$c_p dT - v dp + L_v dr_s \simeq 0 \quad (80)$$

Thus the saturated moist adiabatic lapse rate  $\Gamma_s$  is

$$\Gamma_s = \frac{dT}{dz} = \frac{v}{c_p} \frac{dp}{dz} - \frac{L_v}{c_p} \frac{dr_s}{dz} = \Gamma_d - \frac{L_v}{c_p} \frac{dr_s}{dz} \quad (81)$$

As temperature falls with height,  $\frac{dr_s}{dz}$  is negative implying that  $\Gamma_s > \Gamma_d$ . Recalling that  $\Gamma_d = -9.8 \text{ K km}^{-1}$ ,  $\Gamma_s$  is generally between  $-3$  to  $-4 \text{ K km}^{-1}$  in the lower troposphere. **Q: In the upper troposphere  $\Gamma_s \sim \Gamma_d$ , why is this the case?**

## Isobaric Equivalent temperature $T_{ie}$

Let us take two parcels of air at the same pressure, but that differ in both their temperatures and humidity contents; how can we compare them energetically? One measure is the **isobaric equivalent temperature**,  $T_{ie}$ .  $T_{ie}$  is the temperature that a parcel would have if all of the humidity would condense isobarically. As this is an isobaric process then it is clear that

$$T_{ie} = T + \frac{L_v}{c_p} r_v \quad (82)$$

neglecting the temperature dependence of  $L_v$  as always.

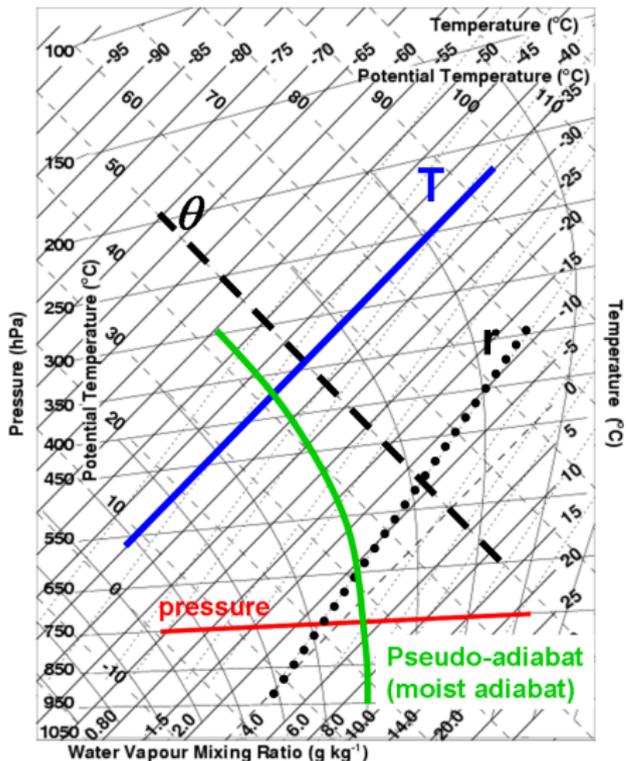
## Adiabatic Equivalent temperature $T_e$

As this isobaric process of  $T_{ie}$  can not occur in the atmosphere, we can instead define the tephigram related **adiabatic equivalent temperature**  $T_e$ , by raising the parcel to the upper-troposphere and then descending to the original pressure.

From (80), we divide by  $c_p T$ , use the first law, and then assume  $T$  is constant in the humidity term (a fairly reasonable approximation as  $T$  varies by at most 30%) will allows us to integrate to get

$$T_e = T \exp \left( \frac{L_v r_v}{c_p T} \right). \quad (83)$$

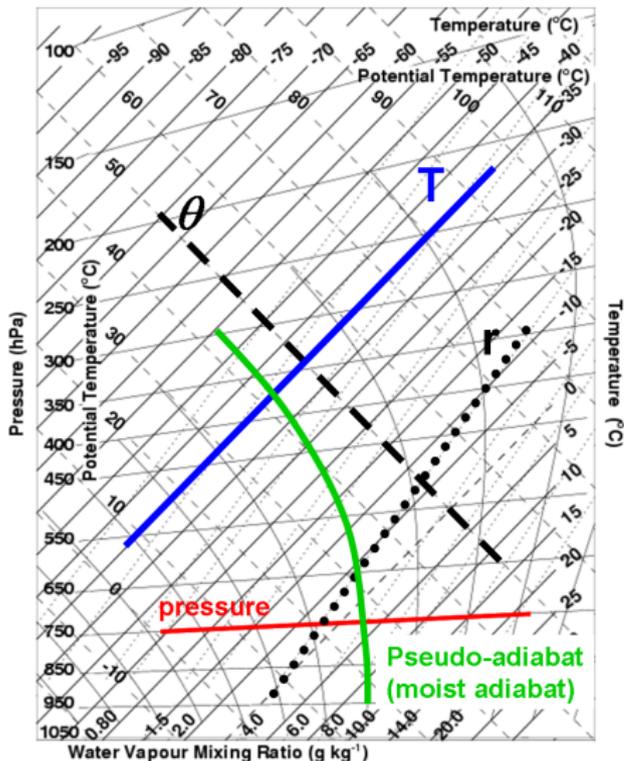
Sometimes you will see  $T$  set to  $T_c$  in the humidity term, giving  $T_c$  in the exponential term on the RHS.



## Equivalent potential temperature $\theta_e$

We found it useful to derive  $\theta_v$  which was conserved in dry adiabatic motions. An analogous quantity (approximately) conserved in moist adiabatic motions is the **equivalent potential temperature  $\theta_e$** .

**Figure:** A blank tephigram with moist

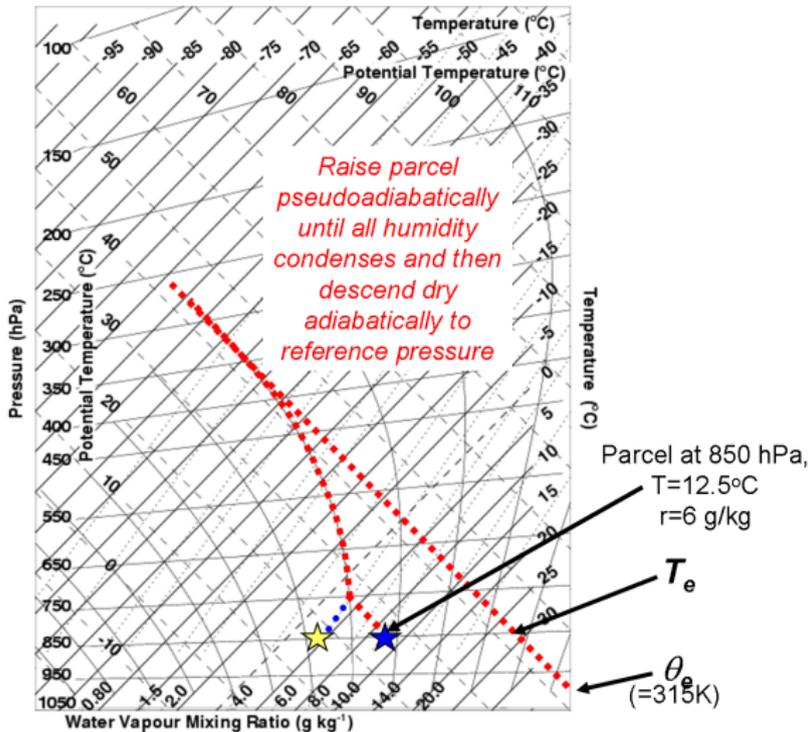


## Equivalent potential temperature $\theta_e$

We found it useful to derive  $\theta_v$  which was conserved in dry adiabatic motions. An analogous quantity (approximately) conserved in moist adiabatic motions is the **equivalent potential temperature**  $\theta_e$ .

We will see that each moist adiabat is uniquely labelled by one value of  $\theta_e$ .

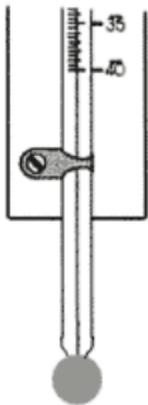
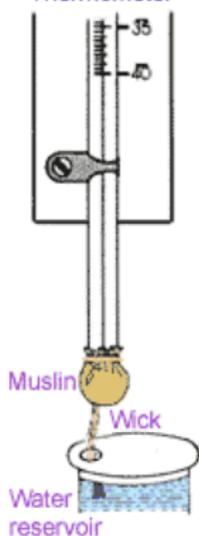
**Figure:** A blank tephigram with moist



The graphical determination of this is given in Fig. 15: after condensation the parcel follows a moist adiabat until all moisture is condensed, and then descends dry adiabatically to the reference pressure.  $\theta_e$  can be approximated by

$$\theta_e = \theta \exp \left( \frac{L_v r_v}{c_p T} \right) \quad (84)$$

**Figure:** A blank tephigram showing the definition of  $T_e$  and  $\theta_e$

Dry bulb  
ThermometerWet bulb  
Thermometer

**Figure:** Method of measuring the wet-bulb temperature

## Evaporation: Wet Bulb Temperature

$$T_w$$

In a Stevenson screen there is usually a wet bulb thermometer to measure the wet bulb temperature  $T_w$ , and determine the atmospheric water vapour content. The thermometer bulb is wrapped in a muslin clothe kept damp by a wick to a liquid reservoir, and the bulb is thus cooled by the evaporation process (Fig. 16).  $T_w$  is the temperature reached if water vapour is evaporated into it until it becomes saturated, with the latent heat provided by the air. Note that  $T_d \leq T_w \leq T$ , and that the difference between  $T_w$  and  $T_d$  provides a measure of the saturation of the atmosphere

To calculate the wet-bulb temperature we consider an air parcel consisting of unit mass of dry air with  $r_v$  mass of water vapour. The heat associated with the evaporation of  $dr_v$  is  $-L_v dr_v$ . Thus

$$(1 + r_v)C_{pm}dT = -L_v dr_v \quad (85)$$

Using the definition of  $C_{pm}$  we get

$$C_p dT \simeq \frac{-L_v dr_v}{(1 + r_v)(1 + 0.9r)} \quad (86)$$

For most purposes, we can neglect the correction factors and assume

$$C_p dT \approx -L_v dr_v \quad (87)$$

Treating  $L_v$  as constant we can integrate to get

$$T_w = T - \frac{L_v}{C_p} (r_s(p, T_w) - r_v) \quad (88)$$

To get  $T_w$  we substitute (53) to give a form that can be solved by iteration:

$$T_w = T - \frac{L_v}{C_p} \left( \frac{\epsilon}{p} A e^{\frac{-B}{T}} - r_v \right) \quad (89)$$

However we can estimate  $T_w$  directly from the tephigram as we shall see now.

Figure 17 shows the construction to derive the wet bulb temperature  $T_w$  from a tephigram by lifting a parcel adiabatic to reach saturation and then following a moist adiabat back down to the original pressure. If the moist adiabat is followed to a reference pressure  $P_0 = 1000$  hPa, then the resulting temperature is known as the **wet bulb potential temperature**  $\theta_w$ .



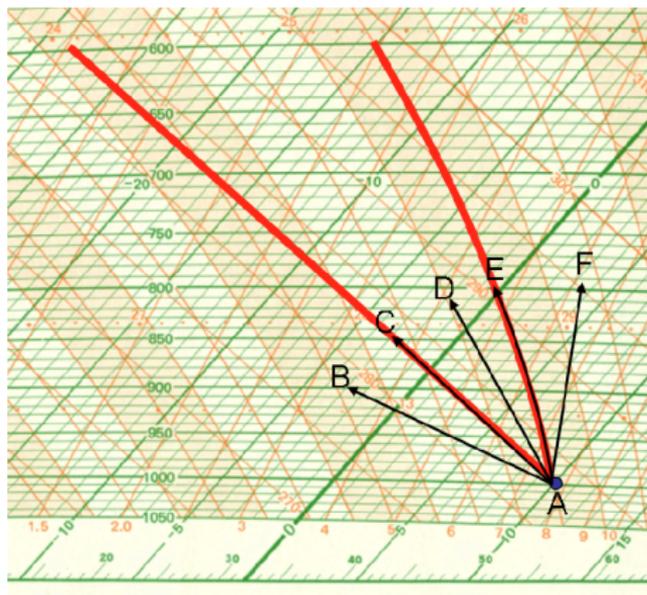
Note that there is a one-to-one unique mapping between  $\theta_w$  and  $\theta_e$ . With each referring to a unique moist adiabat.

## Tephigram Exercises III

Return to the sounding you plotted using the data in Table 2.

- 1 For a parcel of air at 1000 hPa, what is the condensation temperature  $T_c$  and pressure ?
- 2 For a parcel of air at 1000 hPa, what is the dry bulb or dew point temperature  $T_d$ ?
- 3 If rainfall is evaporated into this parcel, at what temperature will it become saturated and what is this temperature called?
- 4 What will be the parcel's mixing ratio at this point?
- 5 What is the parcel's equivalent potential energy  $\theta_e$ ?
- 6 What is  $\theta_e$  for a parcel at 900 and 700 hPa?
- 7 Is  $\theta_e$  increasing or decreasing with height?

The presence of instability is of great importance in weather forecasting and to understand the various climatic regimes. Earlier we classified three stability states in a dry environment. For a moist environment there are five stability categories.

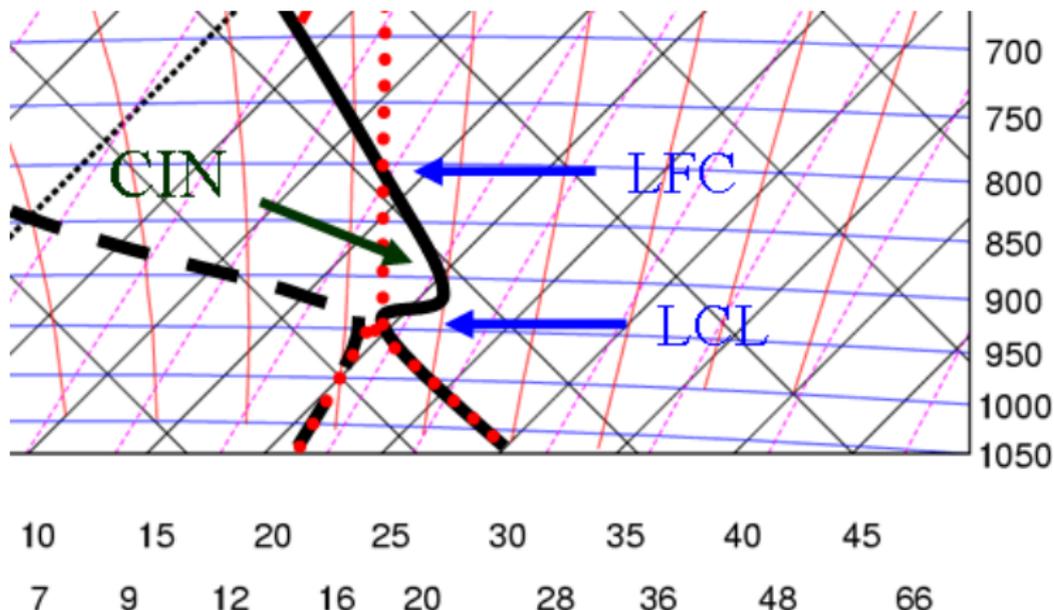


**Figure:** Tephigram showing 5 possible stability categories for environmental lapse rate. The red lines highlight the parcel dry and moist adiabats.

- AB: Absolutely unstable (even if unsaturated, parcel is warmer),
- AC: Dry Neutral (parcel is neutral to dry ascent),
- AD: Conditionally unstable (for unsaturated ascent profile is stable, while it is unstable for saturated ascent),
- AE: Saturated Neutral (for saturated ascent parcel neutral),
- AF: Absolutely stable

## Key convective parameters

We now examine the case of conditional instability further to define a range of parameters that are important for describing the potential for and eventually characteristics of, convection

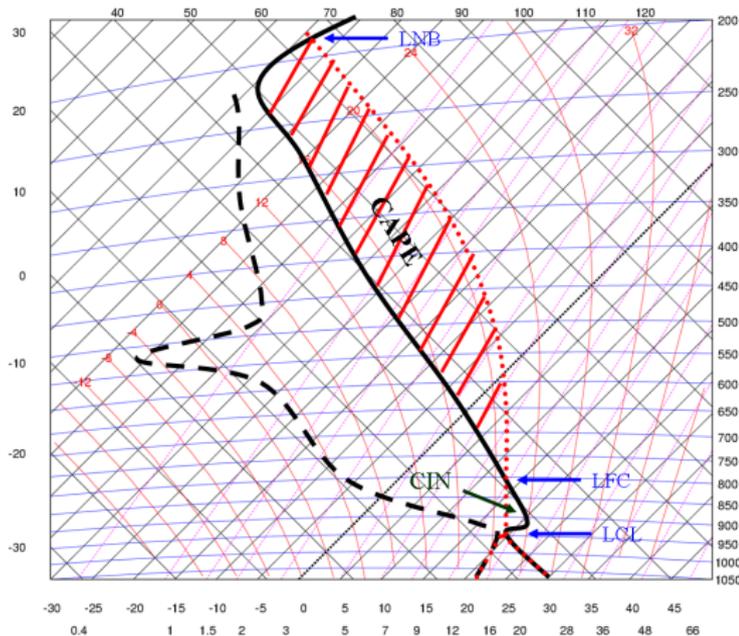


**Figure:** Tephigram showing the boundary layer profiles of temperature and humidity and the pseudo-adiabatic ascent of the surface parcel. See text for details of definitions.

## Convective inhibition, CIN

The area enclosed on the tephigram between the parcel trajectory and the environmental profile when the parcel is negatively buoyant (i.e. colder than the environmental) is equal to the energy that the parcel must be supplied to overcome this inhibition barrier to undergo free convection. This area is thus called the **convective inhibition**, more commonly abbreviated to **CIN**.

After the LFC, the parcel accelerates upwards following a moist adiabat. **Reminders:** What assumptions does this make concerning the parcel? The ascent may continue to the upper troposphere where it recrosses the environmental temperature line. This level is called the **level of neutral buoyancy** or LNB. The parcel will overshoot and undergo oscillatory motion, but the LNB marks the cloud top approximately.



**Figure:** Full tephigram from Fig. 19 additionally showing the full parcel ascent and the positive energy area of CAPE.

## CAPE

The positive area demarked equates to the energy that can be potentially gained by the parcel. We will call the **total<sup>a</sup>** energy available to a parcel starting from level  $i$  the **convective available potential energy** or CAPE.

---

<sup>a</sup>i.e. including CIN. Often the literature refers to the CAPE of a sounding as *only* the positive area, as marked in Fig. 20, but here we follow Emanuel (1994).

If we concern ourselves only with the motions in the vertical then CAPE calculated from level  $i$  is thus the integral of the buoyancy force

$$CAPE_i = \int_i^{LNB} F_B dz, \quad (90)$$

and substituting (39) we get

$$CAPE_i = g \int_i^{LNB} \left( \frac{T_u - T_{env}}{T_{env}} \right) dz, \quad (91)$$

where  $T_u$  is the updraught parcel temperature.

To summarize this, we have defined

- LCL: Lifting Condensation Level
- LFC: Level of Free Convection
- CIN: Convective Inhibition
- LNB: Level of neutral buoyancy
- CAPE: Convective available potential energy

Figure 21 shows the anvil cloud spreading out from a thunderstorm over Mali in West Africa. Notice how flat the anvil top is, marking the LNB. Over the convective updraught region, the cloud top is undulating and higher, revealing the over-shooting updraughts.



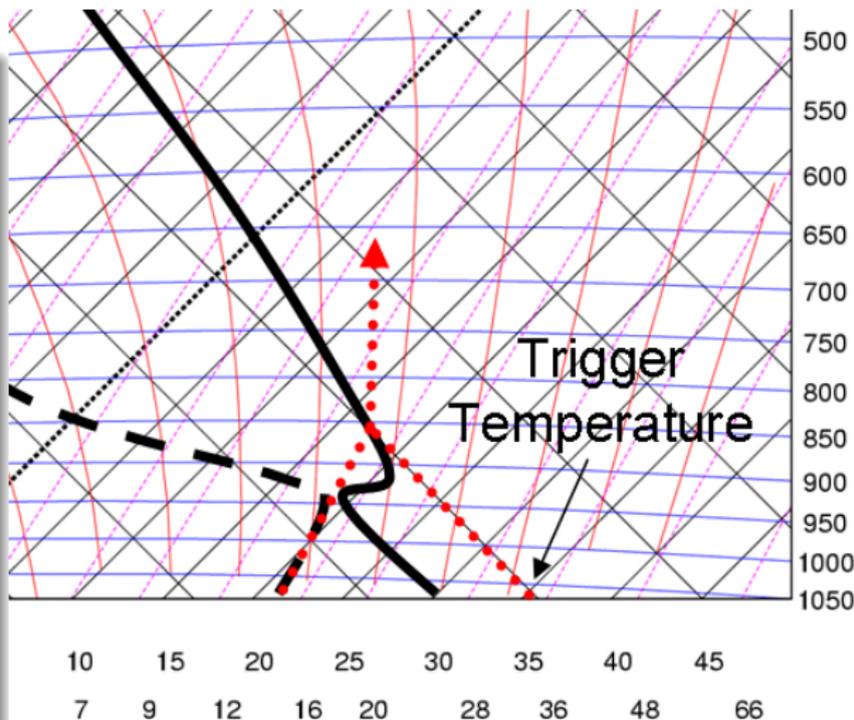
We can see from the tephigram in Fig. 20 that surface parcels are stable to small displacements, but unstable for large displacements. This is why the profile is termed **conditionally unstable** when  $\Gamma_d < \Gamma_{env} < \Gamma$ .

There are a number of mechanisms by which air can be mechanically lifted:

- Flow over orography
- Surface convergence due to low pressure perturbation
- Air lifted over colder, denser air masses at fronts or cold-pools from other convective events.

## Convective Trigger Temperature

The convective trigger temperature is the temperature the surface layer would have to be heated to in order to remove all CIN, assuming no change in boundary layer mixing ratio during this heating process ( **Q: is this a good assumption and when is the trigger  $T$  relevant?** )



**Figure:** Tephigram showing the derivation of the convective trigger temperature

The convective trigger temperature is more relevant for cases where convection is not mechanically forced, for example whether convection will occur in undisturbed conditions as a result of the diurnal heating, particularly over land in the tropics over mid-latitude summers. If the soil conditions are wet then the surface heating can also lead to a significant latent heat (humidity) flux, which will lower the trigger temperature.

## Tephigram Exercises IV

Return to the sounding you plotted using the data in Table Tab:sounding1.

- 1 What is the LCL of a surface parcel of air?
- 2 What is the LFC?
- 3 What is the convective trigger temperature?
- 4 Do you think deep moist convection is likely to occur here?

## Tephigram Exercises V

A 50hPa thick layer of air residing between has the values  $T = 0^{\circ}\text{C}$  and  $r_v = 4.8 \text{ g kg}^{-1}$  at its lower boundary at 700 hPa, and  $T = -2.5^{\circ}\text{C}$  and  $r_v = 3.0 \text{ g kg}^{-1}$  at the upper boundary at 650 hPa.

- 1 Is the layer absolutely stable/unstable, or conditionally unstable?
- 2 The layer is lifted by 100 hPa such that the lower boundary is at 600 hPa, what are the new temperatures at its boundaries?
- 3 Will the lifted layer be unstable to overturning?
- 4 Can you think of a criterion for this layer instability ?

## Convective Instability

If a layer of the atmosphere is lifted that is absolutely stable, but which becomes conditionally (or absolutely) unstable through lifting then the layer is termed **Convectively unstable**

Examining the tephigram it is clear that a criterion for convective instability can be understood in terms of  $\theta_e$  (or equivalently  $\theta_w$ ).

### Three criteria for convective Instability:

$\frac{d\theta_e}{dz} < 0$  : **Convectively Unstable**

$\frac{d\theta_e}{dz} = 0$  : **Convectively Neutral**

$\frac{d\theta_e}{dz} > 0$  : **Convectively Stable**

This kind of layer instability can lead to altocumulus or cirrocumulus clouds, which can be signs of impending arrival of a front, and are illustrated in Fig. 28.

## Conclusions to thermodynamics

In the previous two sections we have introduced variables that described the thermodynamic state of air and allow us to compare the energetics of two air masses equivalently that may have differing properties of temperature and humidity.

## Conclusions to thermodynamics

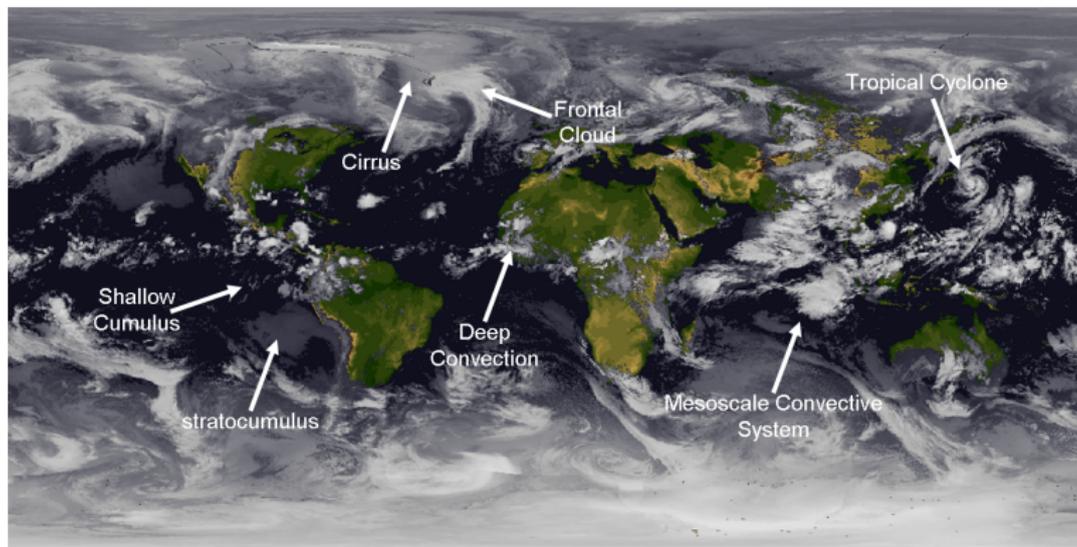
In the previous two sections we have introduced variables that described the thermodynamic state of air and allow us to compare the energetics of two air masses equivalently that may have differing properties of temperature and humidity. We discussed a number of conserved variables, that act as “markers of air” and are conserved under adiabatic motion.  $\theta$  was conserved in dry adiabatic ascent of a dry air parcel, and for a moist air parcel undergoing unsaturated ascent  $\theta_v$  and  $r_v$  were conserved.

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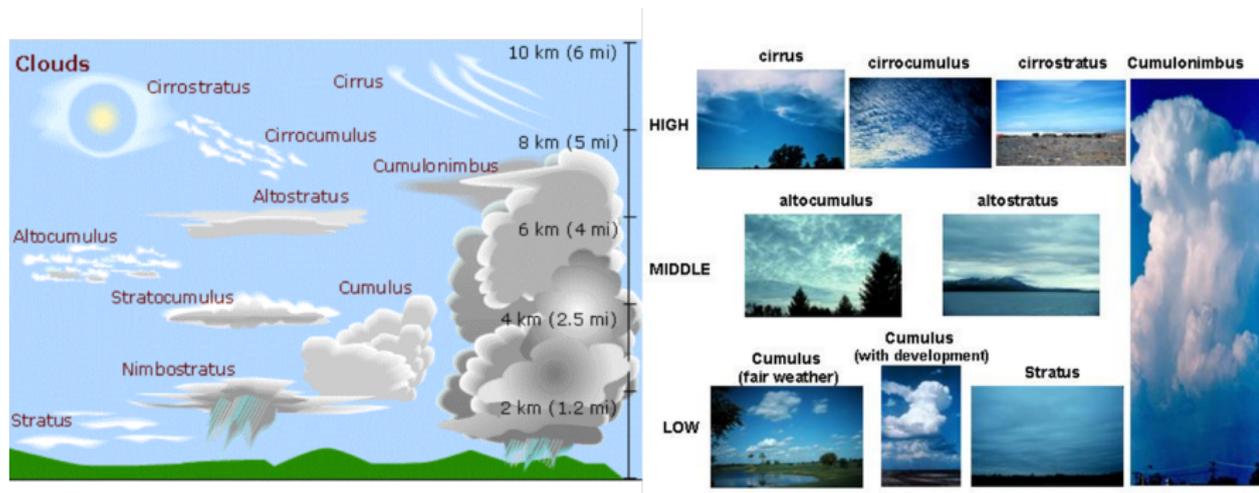
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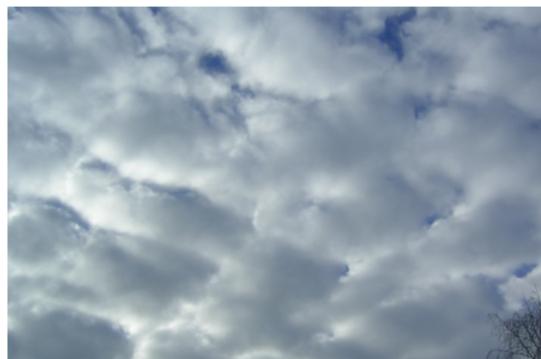
**Figure:** A composite infra-red cloud image where bright white indicates cold cloud tops. Some example convective cloud regimes/types are highlighted

The chart of vertical cloud types (Fig. 24) illustrates common cloud types as a function of height. It is seen that many of these are of type “cumulus” which are associated with vertical atmospheric instability and thus are vertically extended.

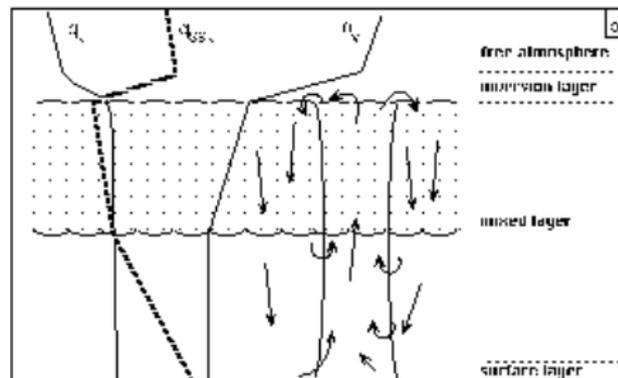


**Figure:** Chart of cloud types as a function of height (left) schematic chart with height scale and (right) photos (source unknown)

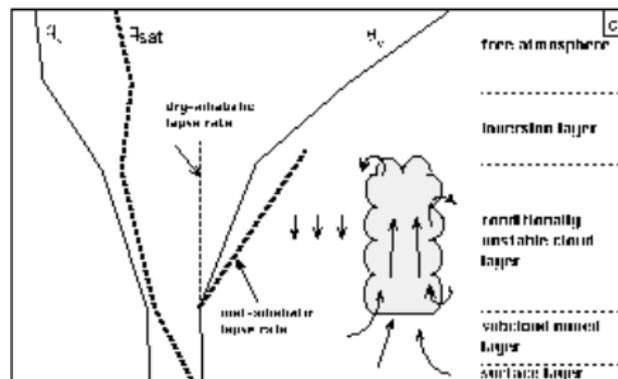
If the turbulent boundary layer becomes deep enough to cause saturation then cloud will form. We saw an example of this fair-weather cumulus earlier in the course. If the well mixed layer is capped by a strong stable temperature inversion then the buoyant turbulent updraughts will be stunted quickly when rising into the base of the stable layer. Thus strong temperature inversions forming under regions of subsidence (such as associated with anti-cyclone or over the Eastern Pacific) lead to stratocumulus with a high cloud cover



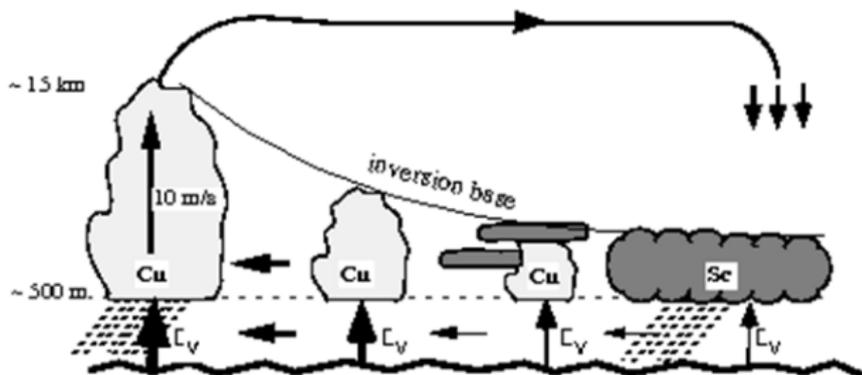
**Figure:** Example stratocumulus



Here we examine the difference in the thermodynamic structures between cumulus and stratocumulus layers



The following schematic shows the large scale circulation associated with the Walker and Hadley cells. Q: why does subsidence lead to a temperature inversion?



**Figure:** Schematic of the Walker/Hadley circulations, showing Stratocumulus forming under the strong capping inversions associated with the subsidence that balances deep convection. (source: S. De Roode cloud lecture notes)

To answer this question we return to the tephigram again and plot the imaginary trajectory of a parcel of air starting in the boundary layer over the Western Pacific (which we will assume is on the left side of the schematic in Fig. 27).

---

<sup>7</sup>hence the schematic in Fig 27 is a poor one since the arrow is horizontal indicating no subsidence.

To answer this question we return to the tephigram again and plot the imaginary trajectory of a parcel of air starting in the boundary layer over the Western Pacific (which we will assume is on the left side of the schematic in Fig. 27. The boundary layer parcel has  $T = 25^\circ\text{C}$  and a  $RH$  equal to 85%. The parcel undergoes deep pseudo-adiabatic convective ascent until it reaches the upper troposphere, and we shall assume it leaves the deep convective cloud (this process is call **detrainment**) at 200hPa. As the parcel is advected across the Pacific it is also descending<sup>7</sup> **Q: Neglecting other processes what would be its temperature and RH when it arrives at  $p=900$  hPa?**

---

<sup>7</sup>hence the schematic in Fig 27 is a poor one since the arrow is horizontal indicating no subsidence.

We can see that the parcel arrives at the top of the boundary layer extremely dry, and also much warmer than the typical surface temperature in the Eastern Pacific.

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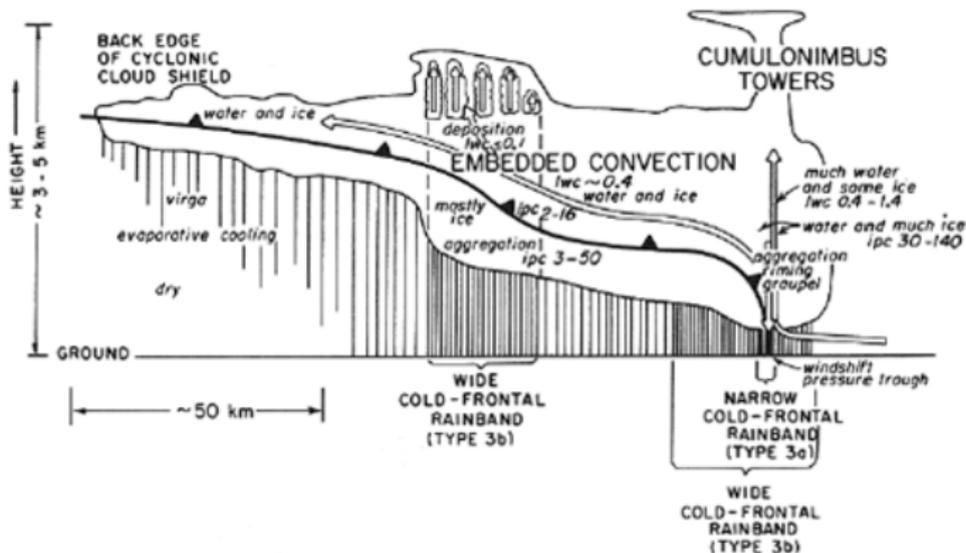
We can see that the parcel arrives at the top of the boundary layer extremely dry, and also much warmer than the typical surface temperature in the Eastern Pacific. However, this simple trajectory model massively exaggerates this, since in reality the air detrained from convection is not 'just' saturated as in the pseudo-adiabatic model, but also contains cloud ice, increasing the total water  $r_t$ , and the air will be subject from moistening by nearby deep convection during its descent. Moreover we know that the warming must be approximately balanced by radiative cooling during its journey across the Pacific. The trajectory plot is also a vast over simplification itself of course, since air will get recirculated in other convective events and so on. The trajectory pattern in the schematic crudely represents an averaged mean flow.

The lifting of convectively unstable layers can lead to the commonly observed cloud types of altocumulus (mid tropospheric, up to around 6000m) and cirrocumulus (upper tropospheric). The vertical division between the two cloud types is not purely taxonomic as cirrocumulus are cloud consisting purely of ice crystals while altocumulus are generally mixed phase clouds (consisting of both liquid and ice). Altocumulus normally occurs when a large air mass is lifted to middle levels by a landmass or an incoming frontal system.



**Figure:** Examples of clouds formed by convectively unstable layers. (a) Altocumulus (b) Altocumulus undulus, resulting from vertical wind shear (c) cirrocumulus (source: [www.theairlinepilots.com](http://www.theairlinepilots.com))

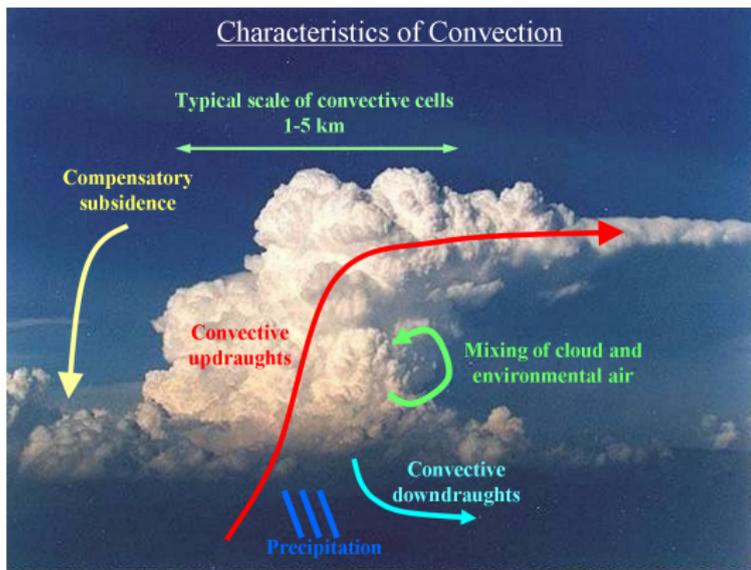
If the unstable layer is deep enough, as is often the case in frontal lifting then the mid and upper level convection can lead to schematic here shows the role that mid or upper level cumulus



**Figure:** Schematic cross section of convection embedded in a front (source).

## Processes in deep convection

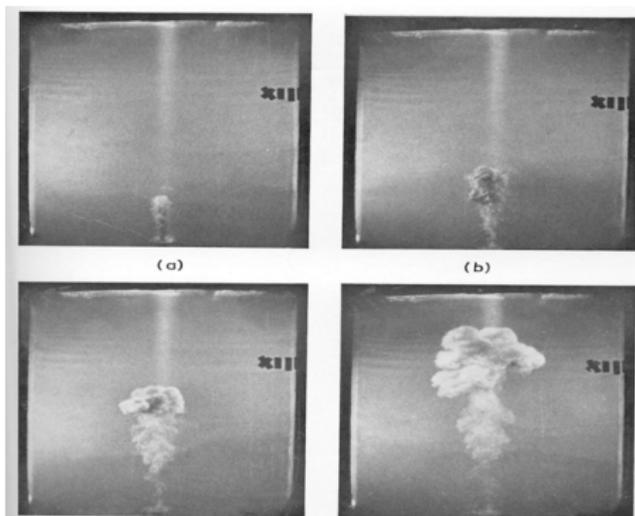
We will now consider some other processes that have been neglected so far in our simple model of deep convection, summarized in Fig. 30.



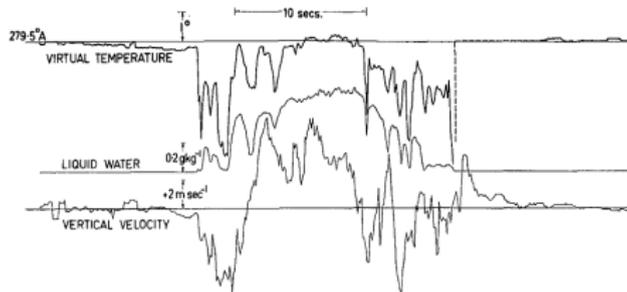
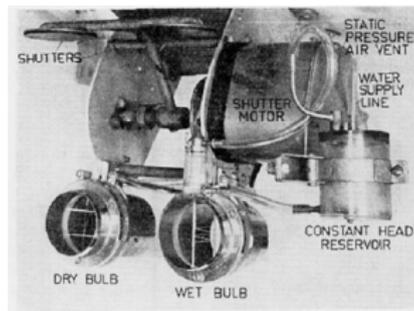
**Figure:** Schematic of other processes to consider in deep convection

We saw from the tephigram study that convective elements can penetrate all the way to the tropopause. However, our model of convection was extremely simple, considering a parcel that undergoes ascent without mixing.

In reality the billowing turbulent motions indicate that mixing of air between the cloud and environment occurs. Indeed the process of mixing was proven in early laboratory studies using buoyant plumes of dyed liquids mixing, however these studies were unable to adequately represent the effect of latent heating. The understanding and representation of mixing in convection is a long-standing problem.



The highly turbulent nature of clouds was evidenced in the first aircraft measurement that were taken after the second world war, such as by (Byers and Braham Jr., 1948; WARNER and Newnham, 1952; Warner, 1955). Here we reproduce slightly later examples from Telford and Warner (1962) we a photograph of the wet-bulb and dry-bulb instruments (Fig. 32). Note the turbulent nature of the cloud and the main updraught is towards the centre of the cloud region while the main (saturated) downdraught areas are at the edge of the cloudy region. This led to the bubble model of cumuli development.



**Figure:** Some of the instrumentation and an example transect through a

## Mixing in Cumulus clouds

Consider two masses of moist air at pressure  $p$  with diverse properties (we use subscripts 1 and 2) of temperature and humidity and are subsequently thoroughly mixed isobarically. The specific humidity of the mixture is

$$q_v = \frac{M_1}{M_1 + M_2} q_{v1} + \frac{M_2}{M_1 + M_2} q_{v2} \quad (92)$$

We can write this relationship approximately using mixing ratios and vapour pressure

$$r_v \approx \frac{M_1}{M_1 + M_2} r_{v1} + \frac{M_2}{M_1 + M_2} r_{v2} \quad (93)$$

$$e \approx \frac{M_1}{M_1 + M_2} e_1 + \frac{M_2}{M_1 + M_2} e_2 \quad (94)$$

We will assume that there is no net loss of gain of heat during mixing so that the change in heat of the two parcels is equal

$$M_1(c_p + c_{pv}r_{v1})(T - T_1) + M_2(c_p + c_{pv}r_{v2})(T - T_2) = 0. \quad (95)$$

If we neglect the contribution of water vapour in the heat capacities then we can see that temperature also approximately is linearly mixed:

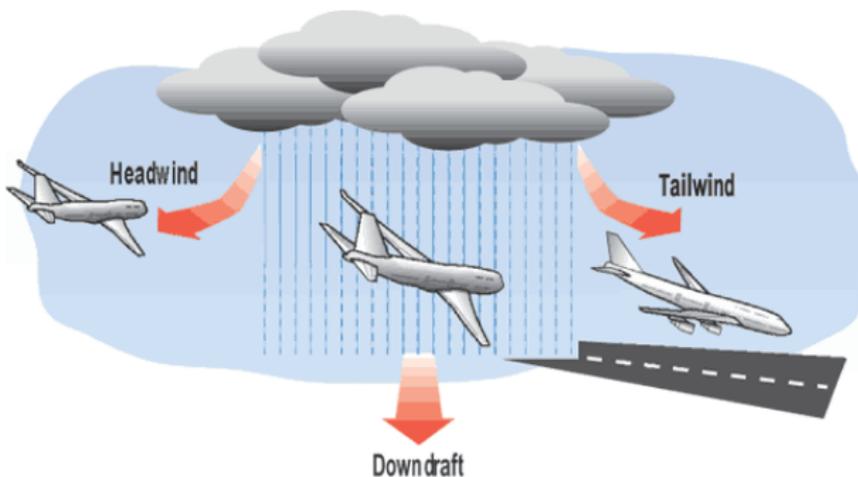
$$T \approx \frac{M_1}{M_1 + M_2} T_1 + \frac{M_2}{M_1 + M_2} T_2 \quad (96)$$

As the saturation vapour pressure is a nonlinear function of temperature, then the approximate linear mixing property of vapour pressure and temperature imply that the mixing of two unsaturated parcels of air can result in a mixed parcel that is saturated (draw sketch). This phenomenon is commonly seen when your breath is visible on a winter's day. In fact it was from this observation that James Hutton deduced in 1784 the concave shape of the Clausius Clapeyron curve a century before it was derived according to ? Middleton(1966).

## Convective Downdraughts

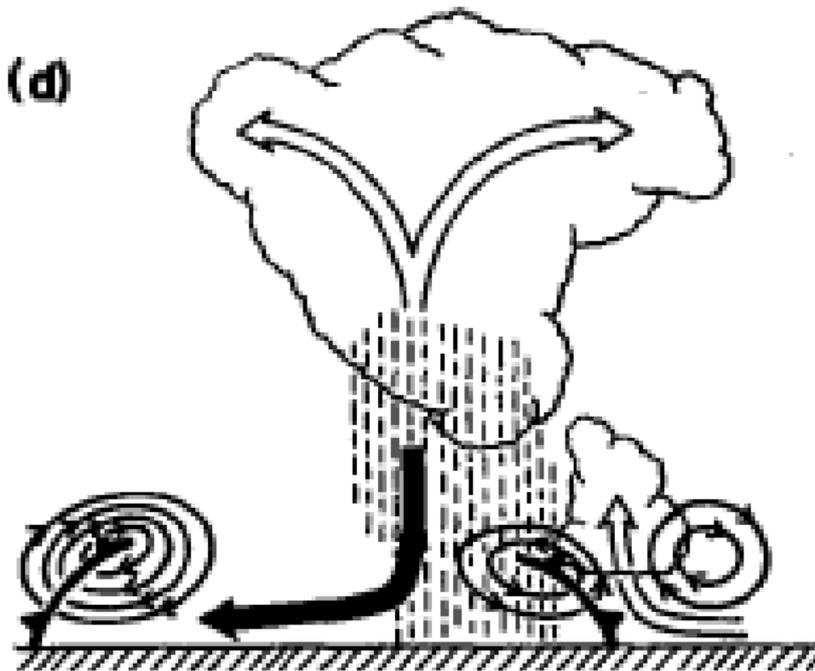
From the transect through a fair weather cumulus cloud it was seen that clouds consist of both updraught and also **downdraught** components. In deep convective systems these saturated downdraughts form at mid-tropospheric levels and can penetrate all the way to the surface, spreading out to form a convective **coldpool**.

Downdraughts and coldpools can be dangerous for aircraft landings (Fig. 33) and thus their short-term prediction is a crucial task of short-term aviation **nowcasting** in potentially convective situations. Severe events are sometimes referred to as microbursts.



**Figure:** Schematic showing how a downdraught or severe microburst can cause an aircraft stall on approach (source: [electronicdesign.com](http://electronicdesign.com))

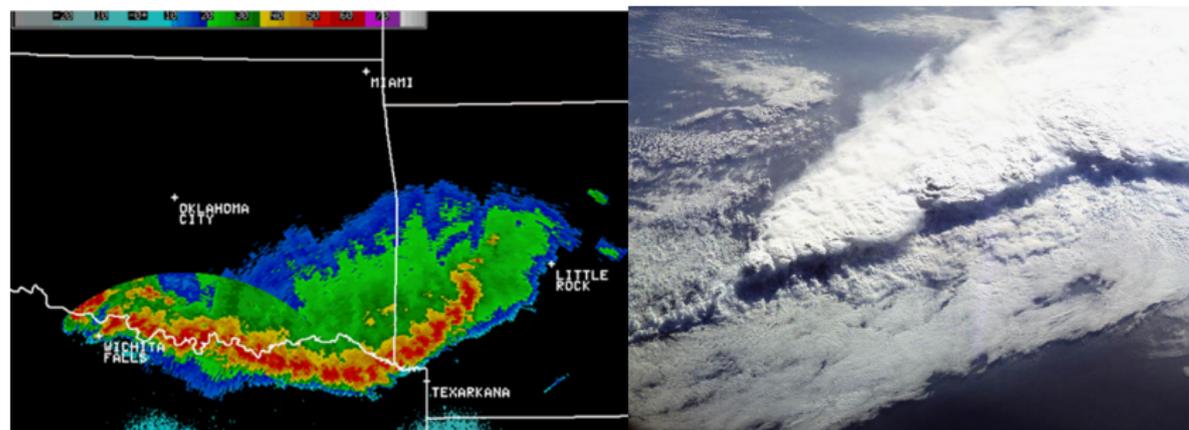
Downdraughts and their associated coldpools are an important component of convective systems since the spreading coldpools can lift environmental air and trigger new convective events, as shown in the schematic of Fig. 34.



## Mesoscale convective systems

Thus convection becomes self propagating and one can speak of organised **mesoscale convective systems** (or MCS) consisting of numerous convective cells. Such systems can have a life time that greatly exceeds the life-span of an individual convective events.

In the presence of wind shear the convective can become linearly organised and is referred to as a **squall line**, with examples shown in Fig. 35. In addition to the saturated convective downdraughts, larger mesoscale downdraught also play a thermodynamical and dynamical role in squall line evolution.



**Figure:** Left: Radar image of a squall line over the United States (source: NOAA). Right: Photo of a squall line over Mexico taken from the Space Shuttle (source: NASA)

## Downdraughts and coldpools

Downdraughts are formed both by the frictional drag of falling precipitation and are also buoyancy driven. The following tephigram example shows how important the mixing and precipitation evaporation (wet bulb) processes are to forming convective downdraughts.

## Tephigram Exercises VI

Plot this (slightly doctored) GATE profile on the tephigram.

GATE Sounding

P (hPa)	T (C)	T <sub>d</sub> (C)
1010	24.5	21.6
950	20.2	19.2
900	17.5	16.1
800	13.2	11.0
700	7.2	5.1
600	0.0	-1.6
500	-7.6	-11.0
400	-17.5	-21.8
300	-34.3	-38.9
250	-43.4	-49.3
200	-50.6	-71.2

- 1 Determine the Lifting condensation level (LCL) for the surface parcel
- 2 Determine the Trigger temperature for the surface parcel
- 3 Draw the temperature profile for a moist adiabatically (no mixing) ascending parcel
- 4 Determine the level of neutral buoyancy (LNB)

An updraught air parcel mixes with the cloud environment in a single entrainment event at  $p = 700\text{hPa}$ , such that the mixture is composed of equal amounts of cloudy and environmental air. Precipitation then passes through the parcel and evaporates, bringing the mixed parcel to saturation.

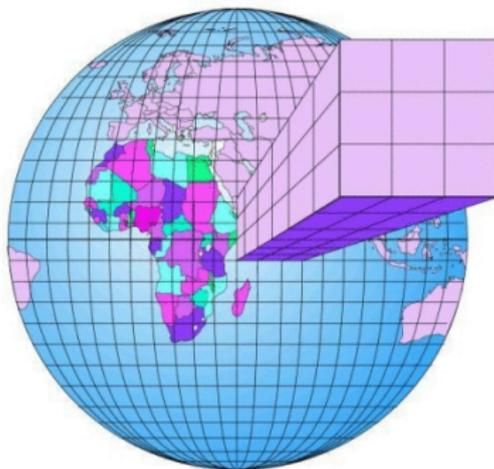
- 1 What is the final temperature of the mixed parcel?
- 2 Can the parcel form a convective downdraught?
- 3 Recalculate this for a mixing event between updraught at 600 hPa.
- 4 If a downdraught can form, we assume it is keep at saturation during its descent by further precipitation evaporation. What temperature does it have at the surface?
- 5 Finally compute the CAPE of the surface parcel (you will probably find it easier to use pressure coordinates for this, assume hydrostatic balance)

## What are numerical models?

In the modelling of the atmosphere, we assume that the behaviour of interest is as if the matter (the gas or liquid) were continuous - We ignore the molecular structure. Thus we introduce the idea of the modelling of a material volume, which has bulk properties of e.g. density  $\rho(\underline{x}, t)$ , temperature  $T(\underline{x}, t)$ , velocity  $V(\underline{x}, t)$  and so on.

A numerical model numerically integrates the equations for momentum, conservation of mass and the thermodynamic equation for a volume of the atmosphere, which is divided into a series of material volumes, called the grid mesh. If we are interested in modelling the atmosphere for a region that is small compared to the earth, we can solve the equations using a Cartesian grid.

For a global the equations have to be transformed into polar stereographic coordinate system, and then the equations will be solved on a grid mesh like that depicted in figure 36.



**Figure:** Schematic of a low resolution grid mesh for a global model.

For a given mesh, we can obviously only model scales of motion down to the size of that mesh. Remember that all fluid in one “grid-box” is considered to be a continuum, and is considered to have one uniform mean property for temperature, velocity etc. The finer the grid-scale we use, the greater the range of spatial scales of motion we can represent, but at the same time the model becomes much more expensive to run.

The use of a finite grid mesh implies that there are some processes that will occur on smaller scales. Q: Can you think what some of these processes may be?



The problem is that these small-scale processes can reflect back on the large-scale equations of dynamics and thermodynamics, and thus can not be ignored. We therefore attempt to construct physical models (that can be simple or very complicated) that use the large-scale grid resolved parameters ( $T, q, u, v, w$  etc) as *input* to determine these subgrid-scale processes (e.g. the strength of turbulence). These models are called **parametrizations** since the process is not explicitly modelled. The ultimate output of the parametrization scheme is the tendency of the the large-scale model equations due to the parametrized process.

To illustrate the concept, let us consider the barotropic vorticity equation; the first numerical weather prediction models were based on this.

$$\frac{\partial \xi}{\partial t} + \nabla \cdot (\underline{v} \xi) + \beta \underline{v} = 0, \quad (97)$$

where  $\xi$  is the relative vorticity and  $\underline{v}$  the horizontal velocity vector and  $\beta$  is the poleward gradient of the Coriolis parameter.

## Reynolds Averaging

Now we will perform Reynolds averaging. Let

$$\xi = \bar{\xi} + \xi' \quad (98)$$

where  $\bar{\xi}$  is the mean of  $\xi$  in our discretised gridbox for a given model timestep, and  $\xi'$  is the *local* deviation from the average. Similarly  $\underline{v} = \bar{v} + v'$ .

Substituting into the vorticity equation we get

$$\frac{\partial \bar{\xi}}{\partial t} + \frac{\partial \xi'}{\partial t} + \nabla \cdot (\bar{\underline{v}} \bar{\xi}) + \nabla \cdot (\bar{\underline{v}} \xi') + \nabla \cdot (\underline{v}' \bar{\xi}) + \nabla \cdot (\underline{v}' \xi') + \beta \bar{\underline{v}} + \beta \underline{v}' = 0 \quad (99)$$

We now average this equation over the gridbox  $(\ )$  and timestep, and thus applying the Reynolds averaging assumptions that (i)  $\overline{A'} = 0$  (ii)  $\overline{\overline{A}A'} = 0$  (ii)  $\overline{\overline{A}} = \overline{A}$  the barotropic vorticity equation reduces to

$$\frac{\partial \overline{\xi}}{\partial t} + \nabla \cdot (\overline{\underline{v}} \overline{\xi}) + \nabla \cdot (\overline{\underline{v}' \xi'}) + \beta \overline{\underline{v}} = 0 \quad (100)$$

We note that the equation for  $\overline{\xi}$  looks identical to that of  $\xi$ , except for the addition term  $\nabla \cdot (\overline{\underline{v}' \xi'})$ . *This term  $\nabla \cdot (\overline{\underline{v}' \xi'})$  represents the unresolved subgrid-scale processes.*

The purpose of parametrization is to give an expression for the subgrid-scale terms as functions of the resolved values.

## Constant $K$ diffusion turbulence parametrization

For simplicity we consider the the subgrid-scale flux in one direction only, the  $x$ -direction:

$$\frac{\partial}{\partial x} \overline{(u'\xi')}. \quad (101)$$

Let us suppose that the effect of fluctuations on the subgrid fluctuations is simply to smooth out fluctuations of  $\xi$  (down-gradient mixing by analogy to molecular diffusion). We could represent this is a simple *parametrization* by

$$\overline{(u'\xi')} = -K \frac{\partial \xi}{\partial x}, \quad (102)$$

where  $K$  is a constant, which we shall call the *diffusion coefficient*.

Thus

$$\frac{\partial}{\partial x} \overline{(u'\xi')} = -K \frac{\partial^2 \xi}{\partial x^2} \quad (103)$$

Note that this parametrization is scale-selective in that the term is larger for smaller wavelengths. This means that if the advected field in question (in this case relative vorticity) is initially smooth, then this term is small, and might appear to be negligible.

However, small scales are generated by the flow (see diagram). This idea can be quantified by looking at enstrophy,  $0.5\overline{\xi^2}$ , as a function of scale (geostrophic turbulent theory) (see diagram). There is a cascade of enstrophy from the forcing scale to smaller scales. Even if the model is stable, the solution is increasingly contaminated by noise on the grid-scale. The effect of the diffusion term is to smooth out this noise.

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It is of course possible to extend the simple constant  $K$  diffusion concept to make  $K$  a function of atmospheric stability for example, and the handout notes now proceed to explain one such approach based on the Richardson number. Due to time limitations, this material is for interest only and does not form part of the syllabus for this course.

## Introduction to cloud physics

In all our discussions so far of convection, we have readily assumed that in updraught motions, once saturation is reached cloud drops readily form. However it is not obvious that this is the case. Indeed, we shall see that this is not the case for the formation of ice crystals.

In this part of the course we move to the small scale physics that occurs on the **microscale** of droplets within clouds, known as **cloud microphysics**. What are the processes that we need to consider in clouds therefore?

## Cloud processes

We will consider the

- change of phase from water vapour to liquid droplets or ice crystals
- transformation of small cloud droplets to larger rain drops
- freezing of cloud droplets
- formation of ice crystals from water vapour
- advection/falling of the larger sized droplets (precipitation)
- evaporation/sublimation of precipitation and cloud

## Cloud particle modes

Condensed droplets of water can obviously have a range of sizes, or droplet radius. However the probability density function describing the droplet sizes is not a smooth function from the very small to the very large (large being a relative term!), but instead we shall see that diverse and discrete cloud processes contrive to produce distinct peaks in the droplets size spectra, termed **modes**. We shall follow the convention of considering each mode as a **bulk quantity**.

We will consider modes of

- cloud drops
- rain drops
- ice crystals
- snow flakes

For ice processes the division is less obvious since ice crystals can form many different shapes or **habits**, with differing radiative and microphysical (e.g. fall-speed) properties despite similar particle sizes which may be treated as a separate category. It is always possible to divide each mode into finer size categories of course, for instance considering small and large cloud ice particles separately. Cloud models that treat many size categories are often termed **bin resolving** as they divide each mode into a number of size **bins**

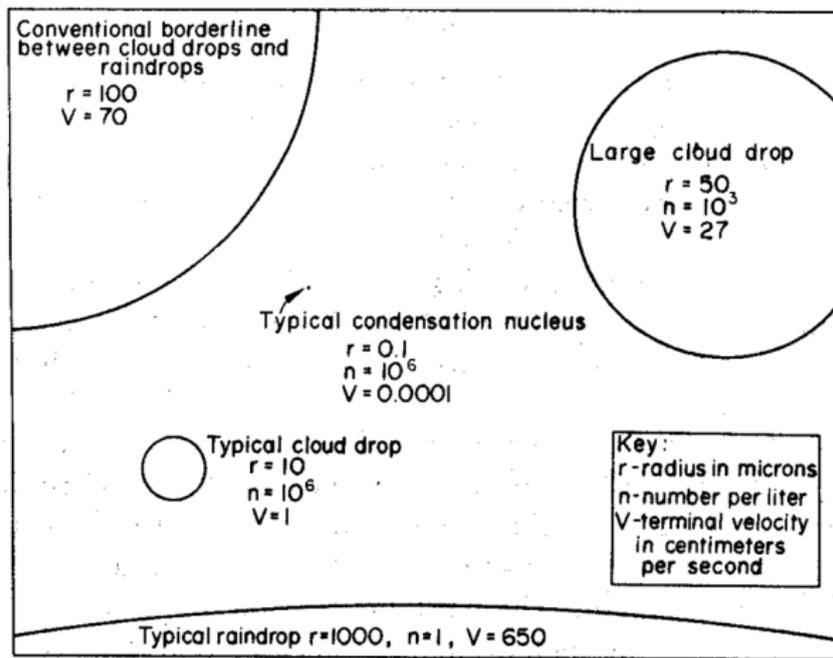
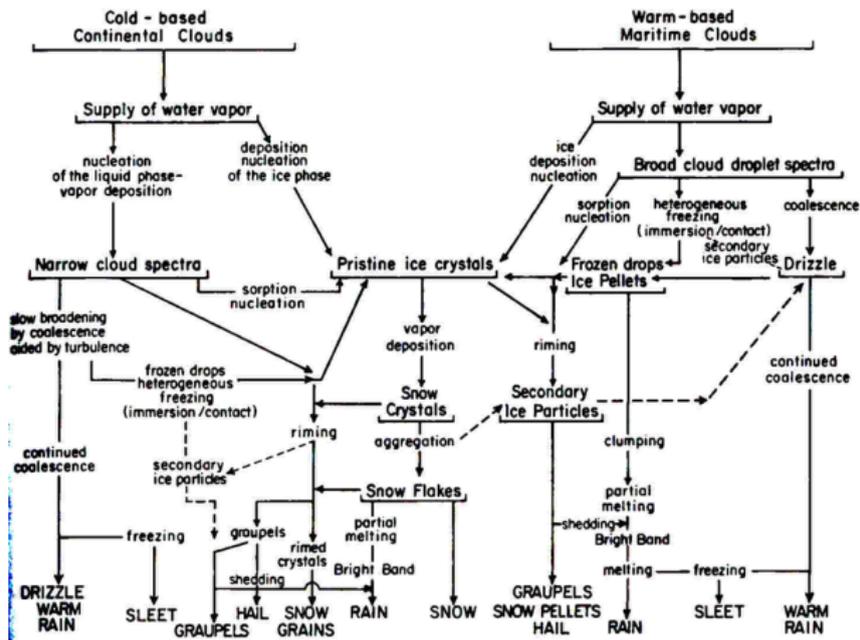


Figure 38 shows a schematic of the typical sizes of cloud particles in **warm phase** (i.e. no ice processes involved) clouds. The cloud droplet has a typical radius of  $10\mu\text{m}$ , (a micron= $1\mu\text{m}$ ), 100 times smaller than a typical raindrop.

**Figure:** Typical drop sizes, from McDonald 1958 ?, reproduced from Rogers and Yau (1989).



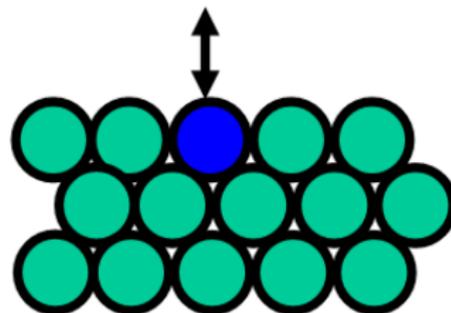
**Figure:** Schematic of cloud process pathways between various cloud particle modes (source unknown)

In general we will consider cloud microphysical processes as pathways that can either convert particles from one or more discrete modes to a different particle mode, or can change the mass or size distribution within one particular mode. Figure 39 reveals a bewildering array of such processes.

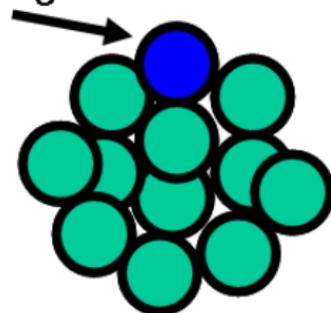
In our earlier lecture we assumed that supersaturated states could not exist and water vapour in excess of the saturation mixing ratio was immediately condensed into cloud droplets. It is not obvious that this should be the case. We will now consider the effects imports for cloud particle activation.

### Saturation over a curved surface

The first aspect is that the relationships given earlier for the saturation vapour pressure was for a **planar water surface**. On the scale of a cloud droplet the curvature of the surface is sufficient to reduce the number of (attracting) neighbouring molecules.



Surface molecule has fewer neighbours



The saturation vapour pressure is thus lower over a **curved surface**, with the saturation vapour pressure of a liquid droplet of radius  $r$  given by:

$$\frac{e_s(r)}{e_s(\infty)} = \exp\left(\frac{2\sigma}{rR_v\rho_L T}\right), \quad (104)$$

where  $\sigma$  is the surface tension, which is the free energy per unit surface area of the liquid and can be viewed as the work per unit area required to extend the surface of liquid at constant temperature<sup>8</sup>.  $\sigma \approx 7.5 \times 10^{-2} \text{Nm}^{-1}$  for usual conditions. This is known as the **Thomson** or **Kelvin** effect.

---

<sup>8</sup>We will use notation  $e_s(r) = e_s^r$  interchangeably. 

We can simplify this to

$$\frac{e_s(r)}{e_s(\infty)} = \exp\left(\frac{a}{rT}\right), \quad (105)$$

where  $a = \sigma/R_v\rho_L$  which is (almost) constant at  $3.3 \times 10^{-7}$  m K. The right hand side of the expression can be usefully approximated to

$$\frac{e_s(r)}{e_s(\infty)} \approx 1 + \frac{a}{rT}, \quad (106)$$

The rate of growth is proportional to the difference  $e - e_s(r)$ . If  $e < e_s(r)$  then the droplet will evaporate, while it will grow if  $e > e_s(r)$ .

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### Homogeneous nucleation of liquid droplets

A chance collision of molecules to form a droplet is called **homogeneous nucleation**. Such an event involving 183 molecules of water vapour forming a droplet of radius of approximately  $10^{-3} \mu\text{m}$  at a temperature of 273K would require a saturation ratio of  $S = 3$ , where  $S$  is the **saturation ratio**,  $S = \frac{e}{e_s(\infty)}$  (so that  $S = 3$  equates to a relative humidity of 300%) to survive and grow.

Such high values are never measured in the atmosphere, thus homogeneous nucleation is not a relevant mechanism for cloud formation. So what is it?

## Aerosols

Aerosols in the atmosphere can range in size from  $10^{-4}$  to  $10 \mu\text{m}$  radius with particle concentrations also widely varying from  $10^3 \text{ cm}^{-3}$  in a remote location to  $> 10^5 \text{ cm}^{-3}$  in an urban environment such as London.

There are a number of **natural and anthropogenic** sources for aerosols:

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- mineral soil dust
- sea salt
- gas to particle conversion  $\text{SO}_2 \rightarrow$  sulphate aerosols
- combustion of fossil fuels / biomass burning

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- mineral soil dust
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- combustion of fossil fuels / biomass burning

Small aerosols with radii  $< 0.2 \mu\text{m}$  are referred to as **Aitken particles**,  $0.2 < r < 2 \mu\text{m}$  are **large aerosols**, and  $r > 2 \mu\text{m}$  are **giant aerosols**. Both **dry deposition** (sedimentation) and **wet deposition** (removal by precipitation) are the sinks of aerosols, with a typical aerosol lifetime being around 1 week.

## Heterogeneous nucleation of liquid droplets

Many aerosols in the atmosphere are hydrophilic and thus water molecules can collect on their surface. Clouds can thus form by a process known as **heterogeneous nucleation**, where water molecules collect on a foreign substance. These hydrophilic aerosols are called **Cloud Condensation Nuclei** or CCN. These CCN are always present in sufficient numbers in the lower and middle troposphere to initiate cloud growth.

CCN aerosols can be insoluble but **wettable**, which means that the surface tension between their nucleating surface and water is sufficiently low and water can form a spherical cap completely surrounding the aerosol. Thus the physics of drop nucleation is the same as for pure water, and only large or giant aerosols ( $r > 0.2 \mu\text{m}$ ) generally have a low enough curvature to form cloud droplets at observed supersaturations.

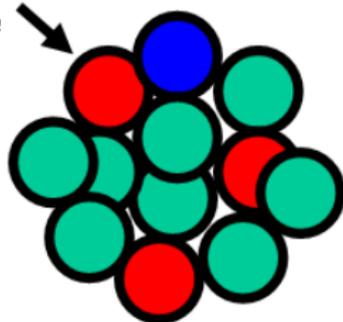
However, aerosols can instead be **soluble**, in which case aerosols with much smaller radii can act as CCN.

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### The solution term

The presence of dissolved substances implies that the some water molecules are replaced from the droplet surface (Fig. 41). Thus the saturation vapour pressure is reduced for a **solute**.

Dissolved substance  
reduces vapour  
pressure



**Figure:** Schematic showing how saturation vapour is instead reduced when there are molecules of dissolved substance (red) present in the droplet.

If  $e_s(sol)$  is the saturation vapour pressure over a solute and  $n_w$  and  $n_s$  are the number of water and solute molecules, respectively, then the fraction of surface which is occupied by water molecules is simply:

$$\frac{e_s(sol)}{e_s^\infty} = \frac{n_w}{n_w + n_s} = \left(1 + \frac{n_s}{n_w}\right)^{-1} \simeq 1 - \frac{n_s}{n_w}, \quad (107)$$

where the final approximation assumes  $n_s \ll n_w$ . In a droplet the number of water molecules is proportional to  $r^3$ , thus for a fixed mass of aerosol this effect adjust the saturation vapour pressure by a factor

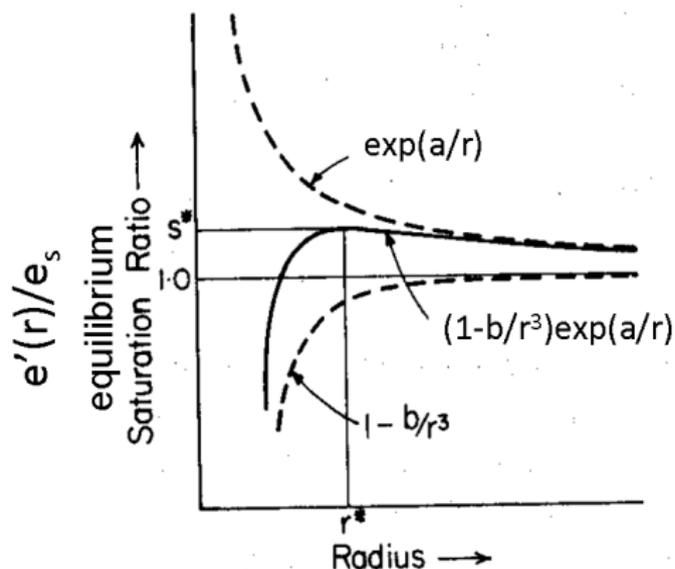
$$\frac{e_s(sol)}{e_s^\infty} = 1 - \frac{b}{r^3} \quad (108)$$

where  $b$  is a constant that depends on the aerosol mass and type.

The curvature (106) and solute (108) effects can be combined to give the resultant equilibrium curve referred to as the **Köhler curve** given by:

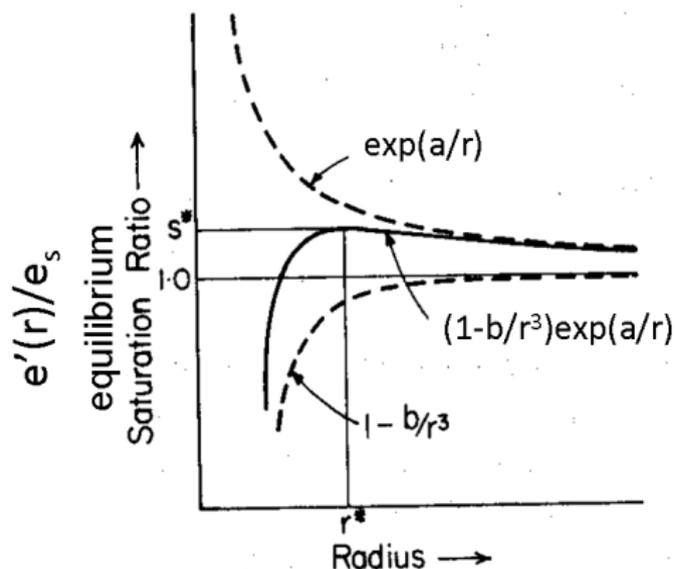
$$\frac{e_s^r(sol)}{e_s^\infty} = \left(1 - \frac{b}{r^3}\right) \exp\left(\frac{a}{r}\right) \approx \left(1 + \frac{a}{r} - \frac{b}{r^3}\right) \quad (109)$$

Figure 42 shows a normalized curve, and the actual shape depends on the mass and type of solute. For a solute formed with  $10^{-16}$  g of ammonium sulphate the solution term is ineffective for radii above  $0.3 \mu\text{m}$ . Due to the  $r^3$  factor the solute term dominates at small droplet radii.



If  $RH$  increases starting from a low value, water vapour start to condense on aerosol parcels as  $RH$  reaches about 80% ( $S = 0.8$ ), referred to as **haze** particles. At these small radii the droplet is stable as an increase in  $RH$  will cause the haze particle to grow until a new equilibrium radius is reached.

**Figure:** Köhler curve for the equilibrium saturation vapour pressure for a liquid solute droplet.



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It is the haze particles that are responsible for reducing visibility on a sunny (humid) day.

In summary, about 10 to 20% of aerosols over oceans can act as CCN, while over land only about 1% can act as CCN.

Nevertheless, the total concentration of CCN is still higher over land with a typical value of  $500 \text{ cm}^{-3}$  compared to  $100 \text{ cm}^{-3}$  over oceans, but these numbers are highly temporarily and spatially variable. Therefore a cloud air parcel brought to saturation over ocean shares the available water between fewer CCN, so we would expect fewer but larger cloud droplets. The consequence of this is that maritime clouds are more likely to rain. We will see that with larger droplets it is easier to grow raindrops.

## Diffusional Growth of droplet

Once a cloud particle is activated it grows rapidly by diffusion of water vapour, and the flux  $F$  ( $\text{kg s}^{-1}$ ) is given by the standard diffusion expression (see sketch):

$$F = 4\pi n^2 D \frac{d\rho_v}{dn}, \quad (110)$$

where  $D$  is the diffusion coefficient ( $\approx 2.2 \times 10^{-5} \text{m}^2 \text{s}^{-1}$  at 273K) and  $\rho_v$  is the vapour density.

In a steady state the diffusion rate is balanced by rate of increase of mass of droplet  $M$ :

$$\frac{dM}{dt} = 4\pi n^2 D \frac{d\rho_v}{dn}. \quad (111)$$

We will assume that the growth is constant. Thus

$$\frac{dM}{dt} \int_r^\infty \frac{dn}{n^2} = 4\pi D \int_{\rho_v(r)}^{\rho_v(\infty)} d\rho_v. \quad (112)$$

giving

$$\frac{dM}{dt} = 4\pi D r (\rho_v(\infty) - \rho_v(r)) \quad (113)$$

We want the rate of change of radius, so we need the expression

$$M = 4/3\pi r^3 \rho_L, \quad (114)$$

which we differentiate to give

$$\frac{dM}{dt} = 4\pi r^2 \rho_L \frac{dr}{dt}. \quad (115)$$

Substituting (115) into (113) gives

$$\frac{dr}{dt} = \frac{D}{\rho_L r} (\rho_v(\infty) - \rho_v(r)). \quad (116)$$

We now use the ideal gas law to change the density to vapour pressure:

$$\frac{dr}{dt} = \frac{D}{\rho_L r R_v T} (S e_s^\infty - e_s^r(\text{sol})) \quad (117)$$

Now we use our earlier approximate expression for the Köhler curve given in (109):

$$\frac{dr}{dt} = \frac{De_s^\infty}{\rho_L r R_v T} \left( S + \frac{a}{r} - \frac{b}{r^3} - 1 \right) \quad (118)$$

For  $r > 1$  micron this simplifies to

$$\frac{dr}{dt} \simeq \frac{De_s^\infty}{\rho_L r R_v T} (S - 1) \quad (119)$$

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This expression is approximate and ignores an important effect. We have neglected the fact that latent heat is warming the droplet, and to be strict the expression should account for the diffusion of heat *away* from the droplet. This “complication” reduces the droplet growth rate very roughly by a factor of 2.

## Availability of water vapour

For a hypothetical cloud parcel there are two main terms that affect the supersaturation (or equivalently relative humidity):

- increase due to the parcel cooling: rate proportional to the parcel velocity
- decrease due to the diffusion process

The evolution of an air parcel containing a spectra of diverse CCN can be calculated numerically using a Lagrangian **parcel model** (see Ren and Mackenzie, 2005, for an example in ice clouds).

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- Growth of cloud droplets from an initial aerosol of  $0.5 \mu\text{m}$  takes only a few seconds explaining well cloud base is *well defined*,
- Calculated drop size spectra from modelling this process are *narrower* than observed (i.e. less size variability). This is because we have neglected the processes of cloud mixing and coalescence of droplets

## Time taken for diffusional growth

We have an expression for the rate of change of  $r$  as a function of time and supersaturation. If we assume  $S$  is time-independent we can find out how long it takes to grow cloud droplets of a certain size by the diffusion process by integrating (119).

This gives (**exercise: check!**)

$$t = \frac{\rho_L R_v T}{2De_s^\infty (S - 1)} (r^2(t) - r^2(0)). \quad (120)$$

If we take an example of  $T=284\text{K}$  and  $r(0)=0.5\ \mu\text{m}$  and  $S - 1 = 0.002$  (a **supersaturation** of 0.2%) then table 3 gives the time taken to grow a cloud droplet of the given sizes

$r\ (\mu\text{m})$	1	5	10	100
$t\ (\text{seconds})$	1	36	150	15000

**Table:** Time taken to grow a cloud drop of given radius by diffusion.

We recall that  $10\ \mu\text{m}$  was a typical cloud radius and thus growth by diffusion *is* fast enough to produce such droplets. However  $100\ \mu\text{m}$  is a small raindrop size, and it is seen that over 4 hours are required to achieve this initial raindrop, far longer than the observed life-cycles of precipitating clouds.

## Terminal velocity

We now introduce the concept of the terminal fallspeed of particles in a simplified way. The terminal velocity is achieved when the force due to gravity  $F = 4/3\pi r^3 g(\rho_L - \rho)$  is balanced by the drag on the droplet.

However the drag strongly depends on the drop size, as seen in Figure 43.

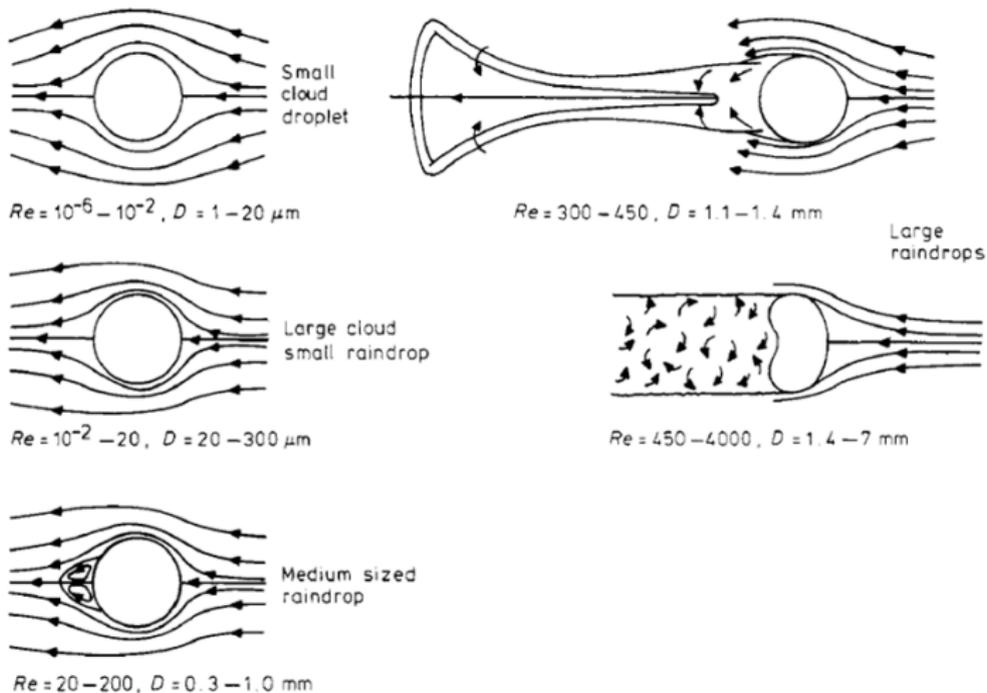


Figure: Changes in the air flow patterns round falling water drops as the

The drag on a droplet is related to the particle velocity  $V$  and the radius  $r$  and can be divided into three regimes, leading to three distinct terminal fall speed  $V_t$  relationships:

- $r < 30\mu\text{m}$ : Drag  $\propto Vr$  giving  $V_t = X_1 r^2$  where  $X_1 \sim 1.2 \times 10^8 \text{s}^{-1} \text{m}^{-1}$ .
- $30 < r < 1000\mu\text{m}$ : Drag  $\propto Vr^2$  giving  $V_t = X_2 r$  where  $X_2 \sim 8 \times 10^3 \text{s}^{-1}$ .
- $r > 1000\mu\text{m}$ : Drag  $\propto V^2 r^2$  giving  $V_t = X_3 \sqrt{r}$  where  $X_3 \sim 250 \text{s}^{-1} \text{m}^{0.5}$ .

Table 4 gives typical fallspeeds as a function of radius.

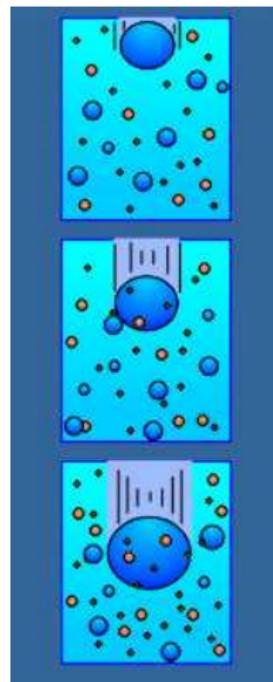
$r$ ( $\mu\text{m}$ )	1	10	20	100	1000
$V_t$ ( $\text{m s}^{-1}$ )	$1.2 \times 10^{-4}$	0.012	0.048	0.8	8

**Table:** Time taken to grow a cloud drop of given radius by diffusion.

From these fallspeeds we notice two facts:

- The fallspeed of typical cloud droplets (2 to 20  $\mu\text{m}$ ) is negligibly small compared to typical updraught and downdraught velocities, with a 10 micron droplet requiring a day to fall 1km. This means that to a good approximation we can assume that cloud droplets are in suspension in the air.

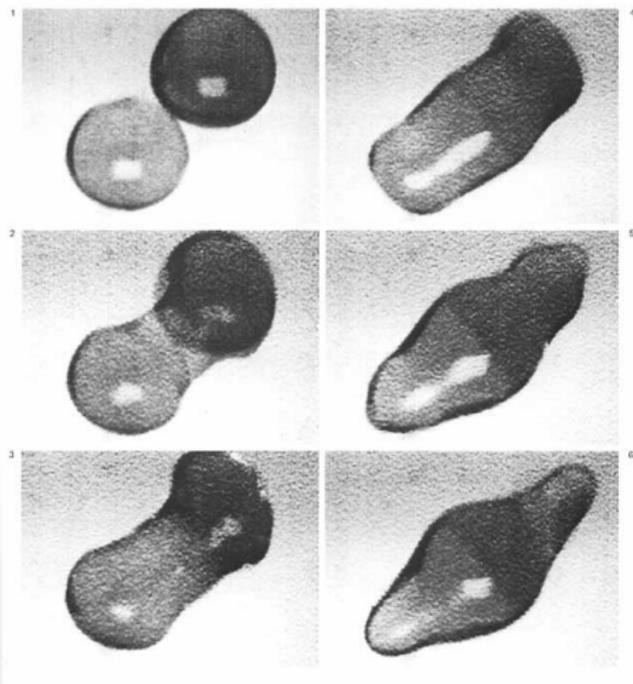
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- The differential terminal fallspeeds also implies that larger droplets falling faster than smaller ones may collide and collect smaller droplets during their descent.



**Figure:** Schematic of larger raindrops colliding with

We consider a large drop of radius  $R$  is falling through a cloud of smaller droplets radius  $r$ . If we take a simple view and assume that any small droplet in the path of the large droplet comes into contact and is **collected** (as in the sequence of Fig. 44, the volume of droplets collected per unit time is  $\pi(R + r)^2(V - v)$ . The liquid water content of the small droplets is  $L = q_l \rho$  in  $\text{kg m}^{-3}$ , giving a mass accumulation rate of

$$\frac{dM}{dt} = L\pi(R + r)^2(V - v) \quad (121)$$



**Figure:** Sequence of shots showing droplet collision and subsequent cohesion (source unknown)

We will simplify the equation by assuming  $R \gg r$  and  $V \gg v$ , and then use (115) which we recall states  $\frac{dM}{dt} = 4\pi R^2 \rho_L \frac{dr}{dt}$  to give

$$\frac{dR}{dt} = \frac{LV}{4\rho_L} \quad (122)$$

If we take the case of the initial growth of small droplets ( $r < 30 \mu\text{m}$ ) then the terminal velocity was given by  $V = X_1 R^2$  giving

$$\frac{dR}{dt} = \frac{LX_1 R^2}{4\rho_L} \quad (123)$$

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$$\frac{dR}{dt} = \frac{LX_1 R^2}{4\rho_L} \quad (123)$$

We recall that the radius rate change due to the diffusive process was proportional to the inverse of radius. This implies that there is a changing balance between the competition of these two processes, with the dominance of diffusion overcome by the importance of the collection process as the radius increases (see sketch)

We can simply integrate (123) to show that for  $L = 10^{-3} \text{ kg m}^{-3}$  a drop can grow from 20 to 30  $\mu\text{m}$  in approximately 10 minutes. We then need to introduce the relationship  $V_t = X_2 R$  into (122) and integrate to calculate that the raindrop can attain a radius of around 300  $\mu\text{m}$  in 20 minutes.

These times appear to be reasonable compared to cloud lifetimes, but we have ignored two effects in this simple view, **Q: Can you think what they might be?**

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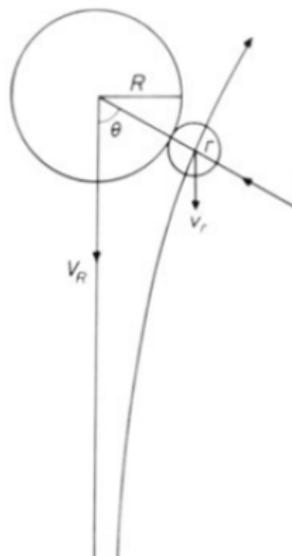
These times appear to be reasonable compared to cloud lifetimes, but we have ignored two effects in this simple view, **Q: Can you think what they might be?** We assumed simplistically that all droplets within the large droplet trajectory collided with the large drop and that all collisions led to coalescence. Neither of these two assumptions are very accurate.

## Collection efficiency $E(R, r)$

We saw earlier in Fig. 43 how the streamlines around various raindrop sizes looked. The lack of inertia of very small droplets implies that they will tend to get swept around the larger droplets if  $r/R$  is small (see Fig. 46). To describe this we introduce the **collision efficiency parameter**  $E(R, r)$ . For small  $r/R$ ,  $E(R, r)$  can be as low as 0.1, while when  $R \sim r$  the flow fields can interact in a complex way and result in  $E(R, r) > 1$ .

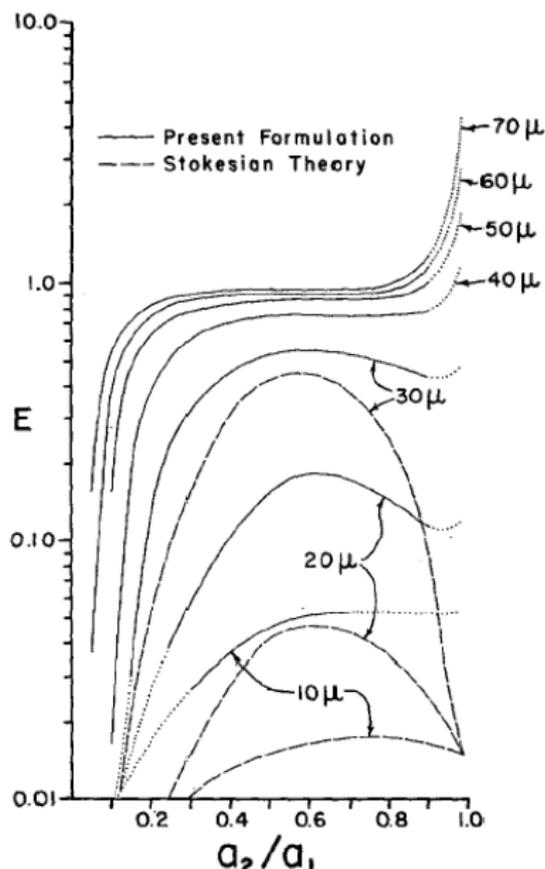
## Coalescence efficiency, $\epsilon$

The **Coalescence efficiency**  $\epsilon$  is often assumed to be  $\sim 1$ , but observations show that it can be lower than 0.5.



**Figure:** Schematic showing more realistic path of small droplet (source Mason (1978))

Figure 47 shows two methodologies for the calculation of  $E(R, r)$  from Klett and Davis (1973) which differ greatly for  $R < 30 \mu\text{m}$  in this case.



Taking the **collision efficiency** and **Coalescence efficiency** into account modifies our radius growth rate equation to

$$\frac{dR}{dt} = \frac{LV\epsilon E(R, r)}{4\rho_L} \quad (124)$$

and growing a droplet from 20 to 30  $\mu\text{m}$  can now take as much as **100 minutes** instead of 10 minutes, and we no longer grow rain drops in the observed time of cloud development.

There are a number of mechanisms that lead to an accelerated growth rate to get us out of this impassé:

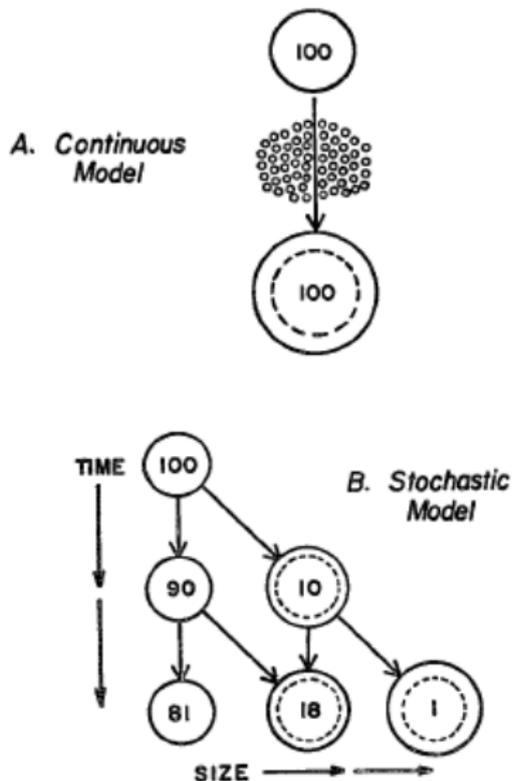
- **Statistical models of raindrop growth**: we assumed that droplet growth was a discrete function of time, whereas it consists of a series of discrete events. If in a time  $\Delta t$  10% of raindrops collide and coalesce then after time  $2\Delta t$  there will be one large drop from 100 initial droplets (see Fig. 48). But this larger droplet will then be favoured for further growth as the growth rate is  $\propto r^2$ . **A spectrum of drop sizes, often bimodal, is therefore generated** (see Fig. 49).

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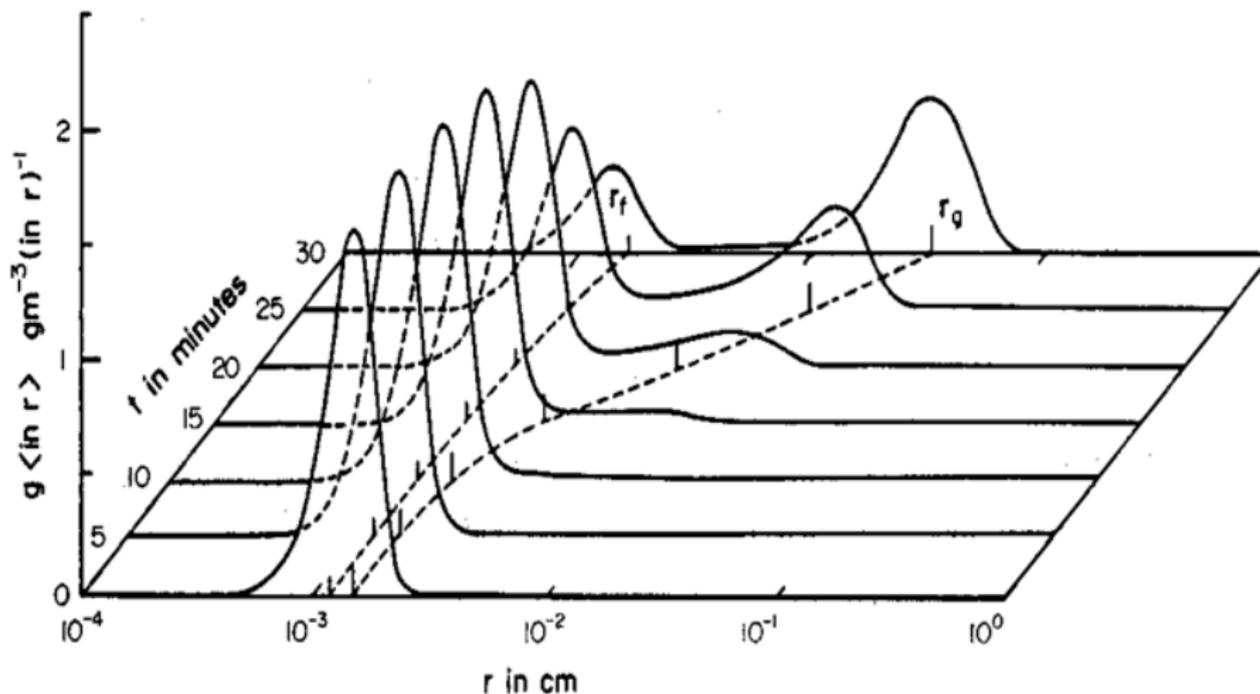
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- It was also assumed that droplets were evenly distributed within the cloud, whereas in reality clouds are **inhomogeneous**, and regions of higher liquid water content will favour droplet growth.
- The process of **entrainment** causes the partial evaporation of droplets broadening the droplet size distribution.



**Figure:** stochastic model of sweep out (source Berry (1967))



**Figure:** Example of the development of a droplet spectrum by stochastic coalescence (source Berry and Reinhardt (1974))

## Introduction to ice processes

The first thing that should be emphasized when considering the ice phase is that **ice phase processes in clouds are far more complicated (Fig. 50) and consequentially poorly understood** relative to the warm phase cloud processes. This course can only skim the surface of the relevant mechanisms.



**Figure:** Example of an ice crystal from <http://www.its.caltech.edu/>)

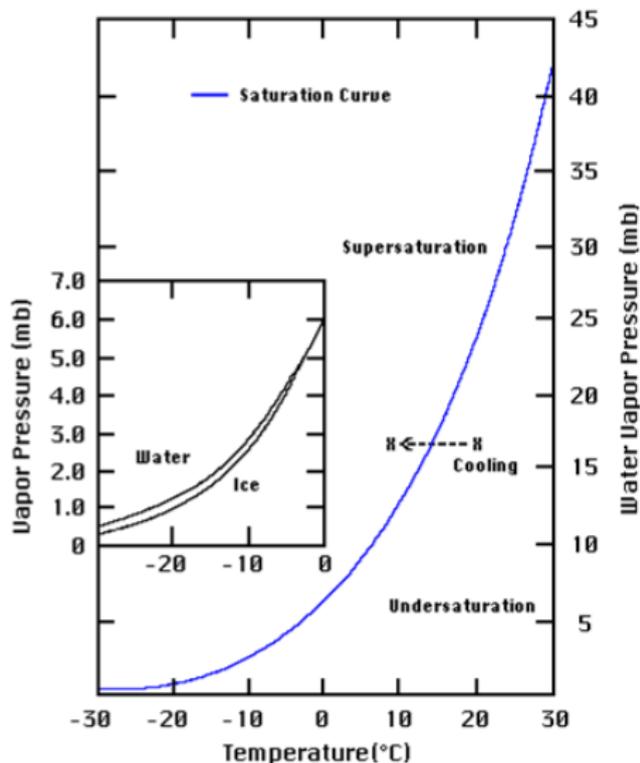
## Saturation over a plane surface of pure ice

Before we continue with the discussion of ice we must first consider the concept of saturation over a planar ice surface. Analogous to the liquid water saturation, the air is said to be saturated if the **deposition** (vapour→ice) rate equals the **sublimation** (ice→vapour) rate.

As the intermolecular bonding energy of molecules in ice is greater than that in liquid water, at a given temperature, the evaporation rate is larger than the sublimation rate. Thus it is clear that  $e_{si}(T) < e_{sw}(T)$  (see Fig. 51) where the saturation vapour pressures are

- $e_{si}$  over ice
- $e_{sw}$  over liquid water.

The vapor pressure of ice and water between  $-30^\circ$  and  $30^\circ$  (mb = millibar). (Berner and Berner 1987)



When the temperature falls below  $0^{\circ}\text{C}$  there is no guarantee that freezing of liquid cloud droplets will occur. Q: Why is it then that ponds freeze over in winter with slightly sub-zero temperatures?

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### Homogeneous nucleation of ice from liquid

Ice particles can be nucleated from either the liquid or vapour phase **in theory**. The homogeneous nucleation of ice from the liquid phase is analogous to the nucleation of liquid drops from vapour. An initial crystal or **ice germ** is formed by statistical fluctuations of the liquid molecular arrangement to form a stable ice-like lattice structure.

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It is clear that a pond therefore freezes more readily than a cloud of liquid droplets as the formation of an ice germ of critical size only has to occur once in the pond, while the event needs to occur in each cloud droplet which is far less likely.

It is clear that a pond therefore freezes more readily than a cloud of liquid droplets as the formation of an ice germ of critical size only has to occur once in the pond, while the event needs to occur in each cloud droplet which is far less likely. Thus it is possible and indeed very common to find liquid cloud drops existing at temperatures much below the freezing point; referred to as **supercooled droplets**.

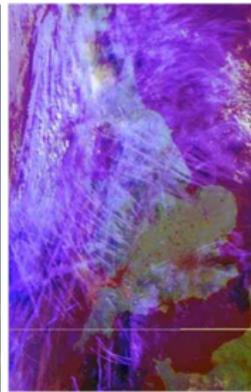
Theoretical results imply that a liquid drop of  $5 \mu\text{m}$  will spontaneously freeze at temperature of around  $-40^\circ\text{C}$ , and measurements inside real clouds show that they contain no ice once temperatures of around  $-35^\circ\text{C}$  to  $-40^\circ\text{C}$  are attained (depending on the cloud drop size spectra). Such clouds are said to be completely **glaciated**.

## Homogeneous nucleation of ice from vapour

It is also theoretically possible to nucleate ice crystals directly from the vapour phase, however theory shows that very high supersaturations ( $> 1000\%$ ) with respect to ice would be required. At these high supersaturations, air would also be supersaturated with respect to liquid water, forming droplets that would freeze. Thus **homogeneous nucleation of ice directly from the vapour phase is not a relevant mechanism** for creating ice cloud.

## Ice supersaturation in the upper troposphere

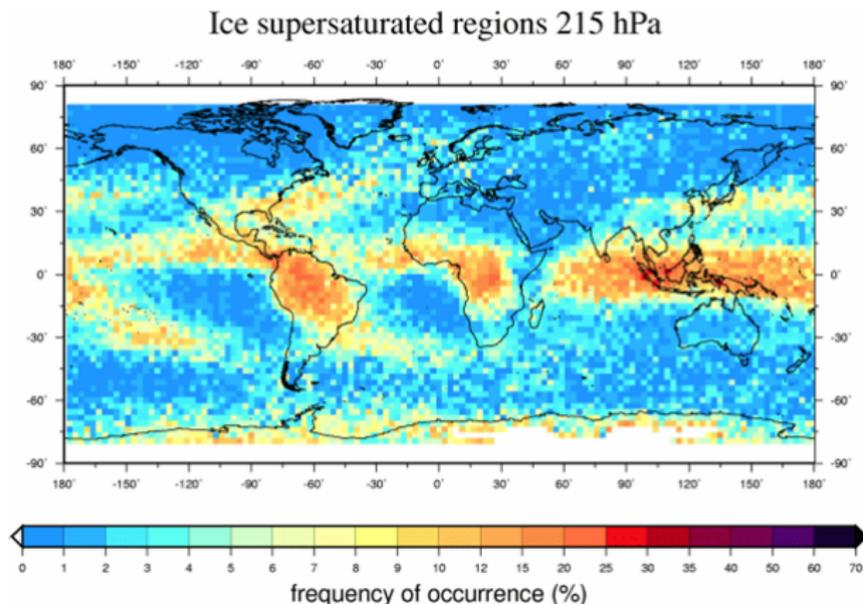
One consequence of the above is that cloud-free air can be supersaturated with respect to ice and not form ice cloud. However there is an upper limit to the amount of ice supersaturation set by  $e_{sw}$  since at this point liquid droplets form which will homogeneously freeze **if** the temperature is below  $-40^{\circ}\text{C}$ . A common sign of the upper troposphere being supersaturated is the presence of **permanent contrail cloud** (Fig. 52).



As temperature get colder,  $e_{sw}$  and  $e_{si}$  diverge, and larger ice supersaturations are possible. In fact it is more complicated and the limit is below  $e_{sw}$  since we recall that aqueous solution droplets (haze) forms at  $RH$  with respect to water substantially below 100%.

Koop et al. (2000) investigates this limit in detail and shows an upper limit of 45% ice supersaturation at  $T = 235$  K increasing to 67% at  $T = 190$  K.

In Fig. 53 Spichtinger et al. (2003) shows using retrievals from microwave limb sounder data that ice supersaturated states are quite common in the upper troposphere.



**Figure:** Frequency of ice supersaturation at 215hPa (source Spichtinger et al., 2003)

We have seen that homogeneous ice nucleation occurs at temperature below  $-40^{\circ}\text{C}$  and that supercooled liquid water is very common below  $0^{\circ}\text{C}$ . However it is also true that ice is observed in clouds between  $0^{\circ}\text{C}$  and  $-40^{\circ}\text{C}$ . Clouds **Q: How could it get there?.**

We have seen that homogeneous ice nucleation occurs at temperature below  $-40^{\circ}\text{C}$  and that supercooled liquid water is very common below  $0^{\circ}\text{C}$ . However it is also true that ice is observed in clouds between  $0^{\circ}\text{C}$  and  $-40^{\circ}\text{C}$ . Clouds Q: How could it get there?.

- Air was previously at much colder temperatures: **parcel history**
- Ice has fallen from above: **ice sedimentation**
- Ice has nucleated in-situ: **how?**

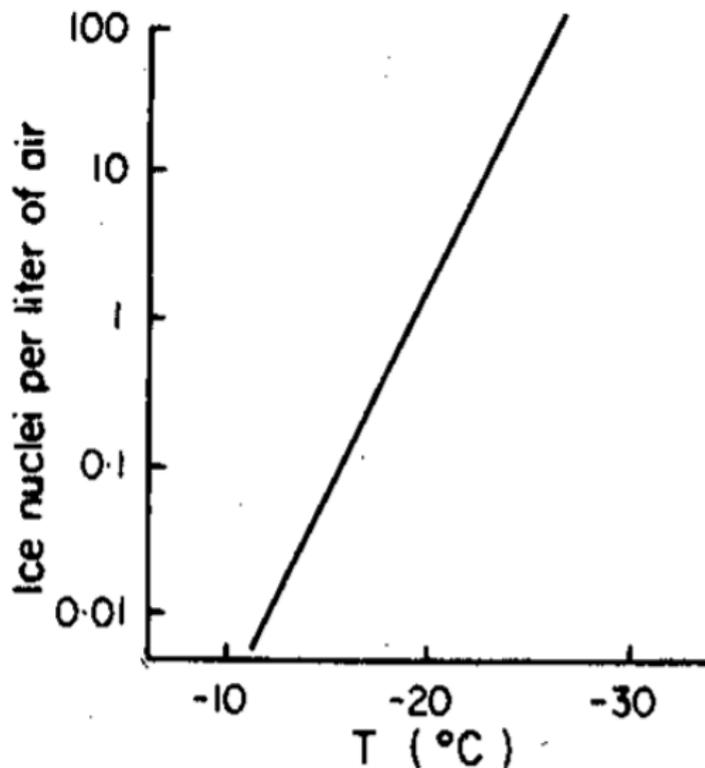
## Heterogeneous nucleation of ice

Aerosols in the atmosphere can also act as **ice nuclei** (IN) if their molecular structure is close enough to the lattice structure of ice. The first point to emphasize is that it is much less common for aerosols to have this property and therefore ice nuclei are much less common than CCN.

At  $T = -20^{\circ}\text{C}$  a typical number concentration (NC) of ice nuclei might be  $10^{-3} \text{ cm}^{-3}$  compared to CCN of roughly  $10^2 \text{ cm}^{-3}$ .

Whether an aerosol can act as a cloud nuclei depends on the **ice supersaturation** and the **temperature**.

As ice supersaturation increases and temperature reduces more aerosols take on the property of being ice nuclei. ? found that for each  $4^{\circ}\text{C}$  of cooling the number of ice nuclei increases by a factor of ten (Fig. 54).



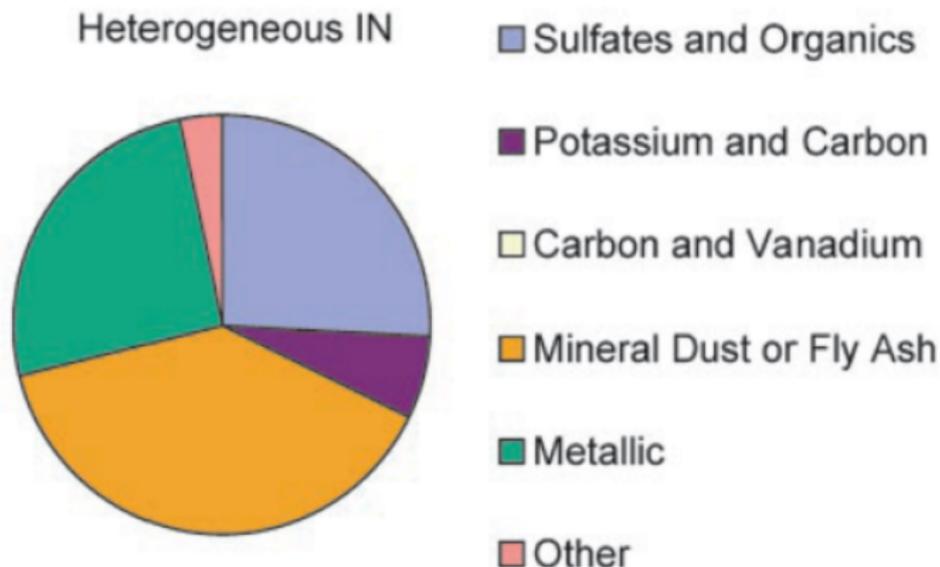
## Common ice nuclei

Some ice nuclei are given in table 5 along with their maximum nucleation temperature.

Substance	Nucleation T (°C)	Notes
ice	0	
silver iodide	-4	used in cloud seeding
clay	-9	often seen in snow crystals
cholesterol	-2	
bacteria(!)	close to 0	used in snow machines

**Table:** ice nuclei and their nucleation temperature maximum

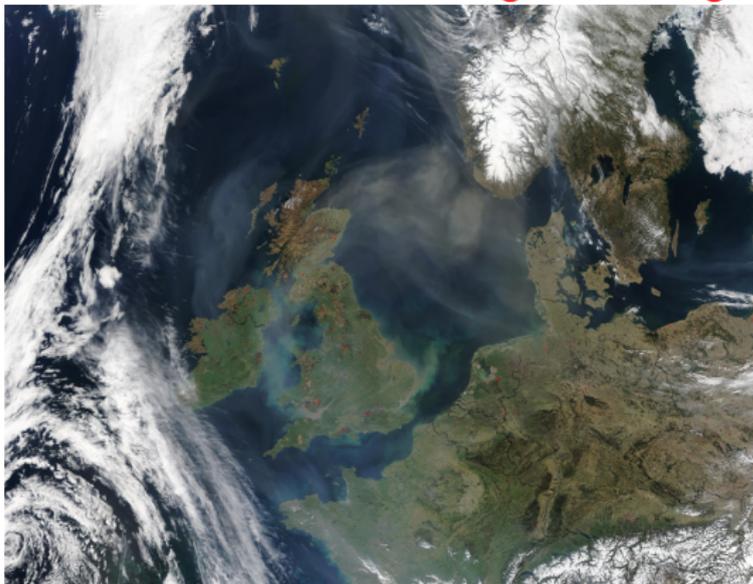
Demott et al. (2003) tested air samples taken from mountain top in mid-Western USA and documented the distribution of IN depicted in Fig. 55.



**Figure:** From Demott et al. (2003), distribution of IN type from air samples taken over the mid-West USA.

It should be emphasized that knowledge of which aerosols can act as ice nuclei is uncertain and the subject of current research. Ice nuclei concentrations are highly variable, and although the Fletcher curve may be accepted as typical, concentrations can vary by orders of magnitude. Aerosols may also be carried a long distance in the atmosphere before being involved in ice cloud nucleation events.

What kind of cloud is this figure showing?



**Figure:** Modis image over Europe (source MODIS website)

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**Figure:** Modis image over Europe (source MODIS website)

Demott et al. (2003) and others have shown that mineral aerosols from Western Africa are a significant source of ice cloud nuclei over the United States for example! In fact many snow or ice particles contain clay mineral aerosols indicating that the desert regions of the world are in general an important source of IN (see Fig. 56).

## General properties of ice nuclei

Observations and laboratory experiments indicate that aerosol particles usually satisfy a list of criteria if they are to serve as IN (Pruppacher and Klett, 1997).

- **Insolubility criterion:** In general, IN must be very insoluble. The disadvantage of a soluble substrate is that it disintegrates under the action of water. Hence the molecular structural requirement for ice nucleation can not be maintained.

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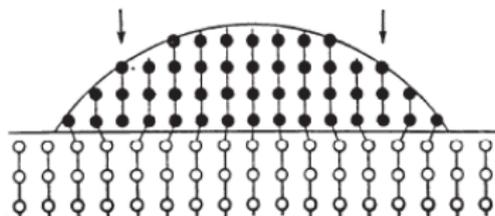
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- **Insolubility criterion:** In general, IN must be very insoluble. The disadvantage of a soluble substrate is that it disintegrates under the action of water. Hence the molecular structural requirement for ice nucleation can not be maintained.
- **Size criterion:** IN must have a size comparable to, or larger than, that of a critical ice embryo. Generally speaking the radius of the IN must be greater than  $0.1 \mu\text{m}$ ; i.e. Aitken particles do not tend to be ice nuclei. At sizes less than this critical radius ( $0.1 \mu\text{m}$ ), the nucleating ability of the particle decreases rapidly (and becomes increasingly more temperature dependent).

- **Chemical Bond criterion:** The chemical nature of an IN, that is the type and strength of the chemical bonds at the IN surface, affect nucleation. Since an ice crystal lattice is held together by hydrogen bonds it is very reasonable to assume that an IN must have similar hydrogen bonds available at its surface for nucleation to occur.

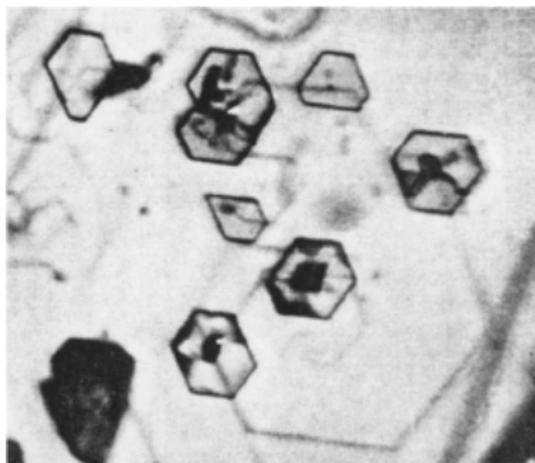
In view of this bond criterion, certain complex organic molecules (i.e. aerobic bacteria) exhibit good ice nucleation abilities. The hydrogen bonding groups in ice are similar to the hydrogen bonding groups in many organic molecules.

- **Crystallographic criterion:** Since ice nucleation on a substrate is actually an overgrowth of ice on the substrate, it is reasonable to expect the nucleating ability of the substrate to increase when the lattice structure of the substrate is similar to the hexagonal lattice structure of ice. In this way, molecular matching between the molecules of ice and the substrate may be achieved (Fig. 57). The crystallographic matching reduces the “misfit” and **elastic strain** in the ice.



**Figure:** An ice-embryo (dark points) growing upon a crystalline substrate with a misfit of 10%. The interface is dislocated, dislocations being indicated by arrows. (from Fletcher, 1969) ?

- **Active Site criterion:** Ice nucleation generally occurs at distinct sites on the substrate. Laboratory studies have indicated that ice nucleation is preferred on those substrates which have pits and steps on their surfaces. Those nuclei that have smooth surfaces are less efficient nuclei, that is, nucleation occurs at **lower temperatures**. An example is shown in Fig. 58 where ice crystals preferentially formed on steps etched onto the surface of a cadmium iodide crystal.



## Mechanisms for Heterogeneous Nucleation

There are several mechanisms for ice crystal nucleation, schematically illustrated in Fig. 59. These are

- heterogeneous deposition nucleation
- condensation nucleation
- contact nucleation
- immersion nucleation

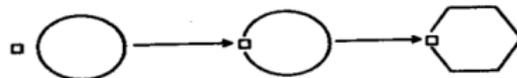
**HETEROGENEOUS DEPOSITION**



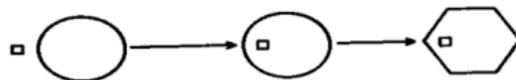
**CONDENSATION FOLLOWED BY FREEZING**



**CONTACT**



**IMMERSION**

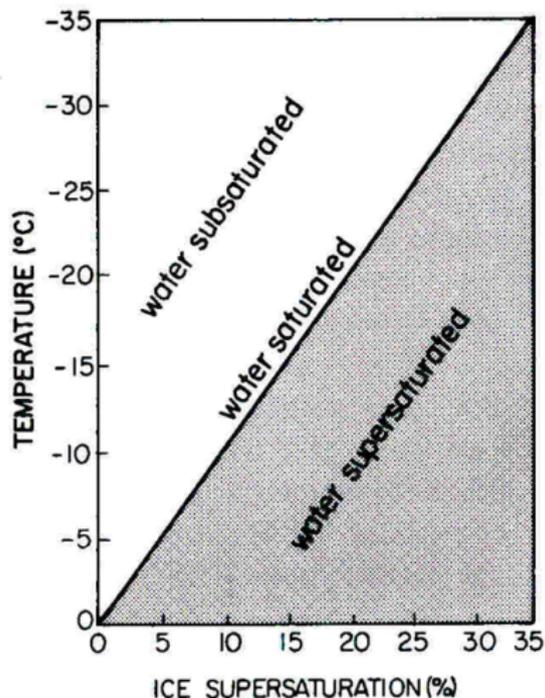


**ICE NUCLEATION MECHANISMS**

**Figure:** Schematic of heterogeneous nucleation pathways (source: Rogers and Yau, 1989).

We divide the consideration of ice crystal growth by diffusion of water vapour into two regimes: **mixed phase** and **glaciated** clouds.

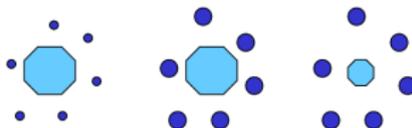
For **mixed phase clouds** we recall that at subfreezing temperatures the ice and water vapour saturation pressures diverge, and thus an air volume at water saturation is supersaturated with respect to a planar ice surface, reiterated in Fig. 60.



## Bergeron-Findeison Effect

The **Bergeron-Findeison effect** is an important growth enhancing mechanism in mixed phase clouds. In a supercooled liquid water cloud imagine one liquid droplet freezes (Fig. 61):

- The cloud is initially saturated **with respect to liquid water**.
- It is thus **ice** supersaturated.
- The ice crystal will grow by diffusion of water vapour towards the crystal,
- This reduces the vapour pressure below the liquid water saturation value.
- The liquid droplets evaporate on a fast timescale maintaining the vapour pressure close to the liquid water saturation.

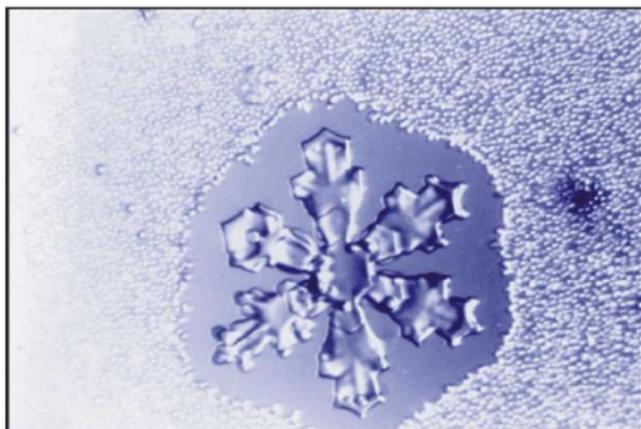


The outcome of the Bergeron-Findeison effect is that the growth rate of the ice crystal is faster than it would be in the absence of liquid cloud droplets. For a water saturated environment the growth rate is fastest when the difference between the saturation limits is largest **in absolute (not relative!) terms**. This occurs around  $T = -15^{\circ}\text{C}$ .

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A laboratory photo shows the result of the BF-effect, with the lack of liquid droplets surrounding the ice crystal on the plate (Fig. 62)

EFFECT OF PHASE DIFFERENCE



## Ice Habits

Ice habits can be very complex and depend on the temperature and ambient ice supersaturation at which ice nucleation takes place (Fig 63). The crystal shape has a strong impact on radiative properties (Macke et al., 1996) and the fall speeds of the ice particle.

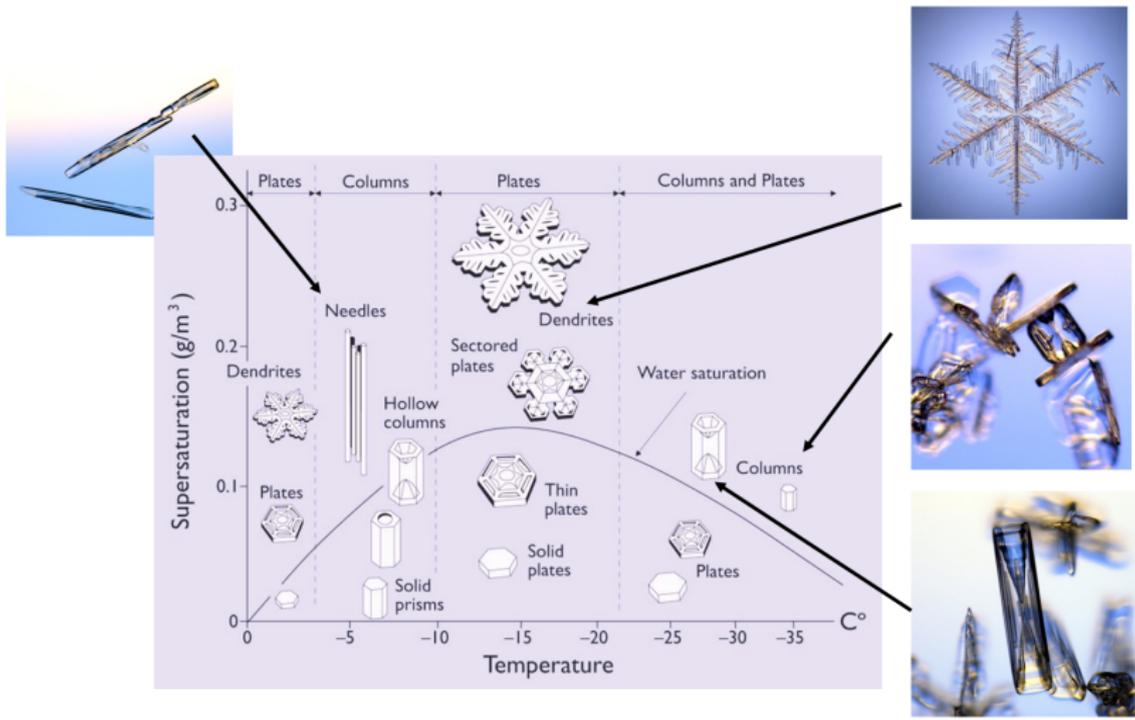
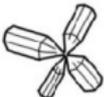
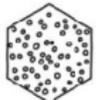


Figure: Ice crystal types as a function of  $S_i$  and  $T$  (source

				
Simple Prisms	Solid Columns	Sheaths	Scrolls on Plates	Triangular Forms
				
Hexagonal Plates	Hollow Columns	Cups	Columns on Plates	12-branched Stars
				
Stellar Plates	Bullet Rosettes	Capped Columns	Split Plates & Stars	Radiating Plates
				
Sectored Plates	Isolated Bullets	Multiply Capped Columns	Skeletal Forms	Radiating Dendrites
				
Simple Stars	Simple Needles	Capped Bullets	Twin Columns	Irregulars
				

## Competition between ice nucleation mechanisms

At temperatures warmer than  $-40^{\circ}\text{C}$  the only nucleation mechanism is heterogeneous. However at temperatures colder than this both heterogeneous and homogeneous nucleation can occur contemporaneously.

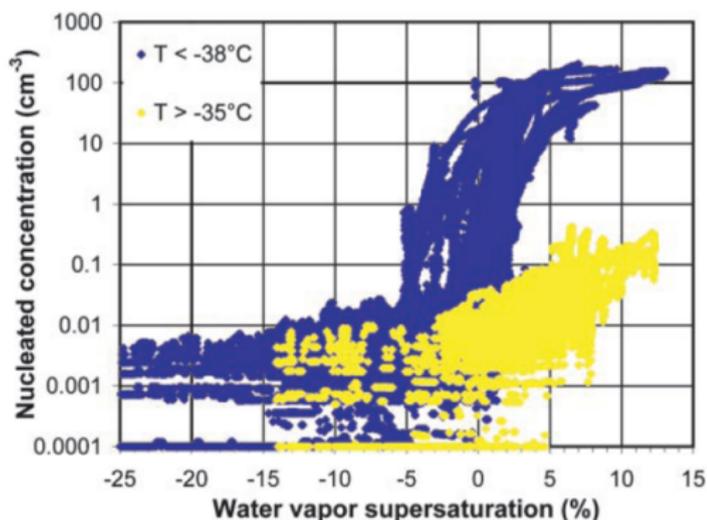
## Competition between ice nucleation mechanisms

At temperatures warmer than  $-40^{\circ}\text{C}$  the only nucleation mechanism is heterogeneous. However at temperatures colder than this both heterogeneous and homogeneous nucleation can occur contemporaneously.

As many IN become active at ice supersaturations of 10 to 30%, well below the threshold range for homogeneous nucleation, then homogeneous nucleation has the potential to be the dominant nucleation mechanism, **if and only if** IN are present in sufficient numbers to prevent the  $RH$  reaching the homogeneous nucleation threshold in a rising (cooling) parcel of air.

**Q: Both mechanism create ice crystals, so why do we care which one dominates?**

Demott et al. (2003) (Fig. 65) documents the concentration of nucleated ice crystals as a function of water saturation for air samples collected over the western United States. For  $T > -35^{\circ}\text{C}$  the NC increases with supersaturation as more aerosols become active. However at cold temperatures there is a step jump close to water saturation to much higher concentrations due to homogeneous nucleation.



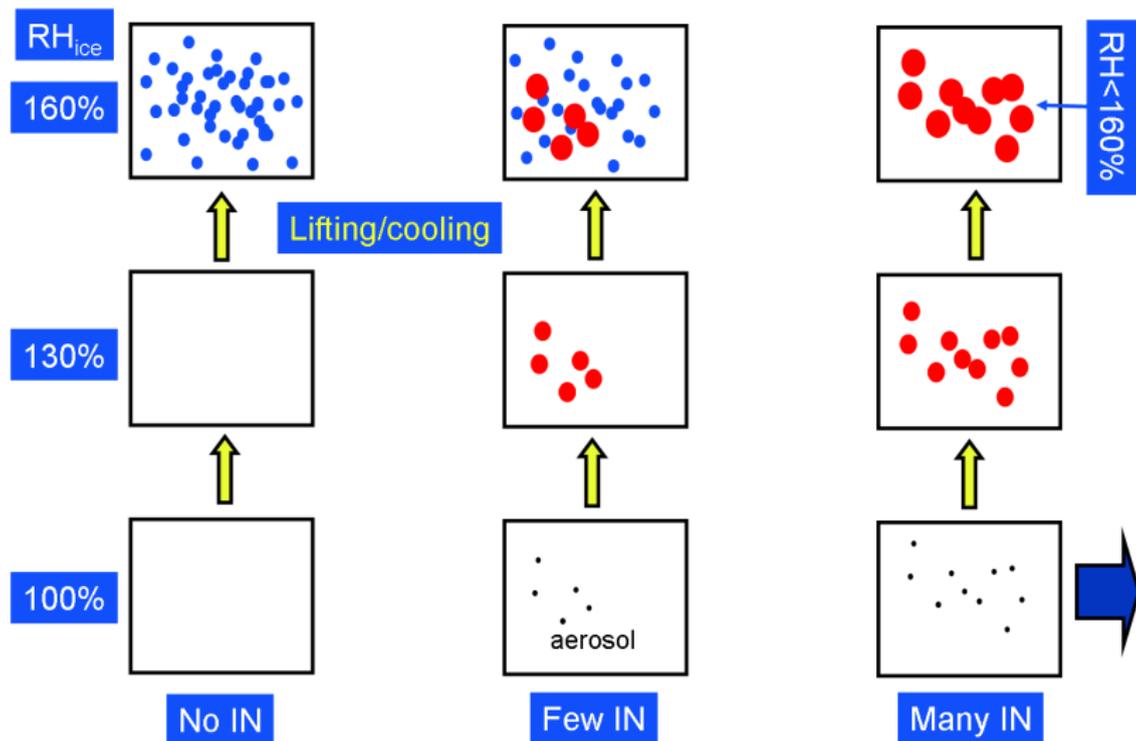
**Figure:** Nucleated ice crystal number in air samples as a function of water saturation at two temperatures. (source: Demott et al., 2003).

Homogeneous nucleation thus has the potential to create a much higher concentration of cloud ice crystals - Q: Why?

The homogeneous nucleation process creates more ice particles since it is due to the freezing of aqueous solution droplets and thus related to the number of CCN. The particles nucleated by homogeneous process are much smaller in size therefore.

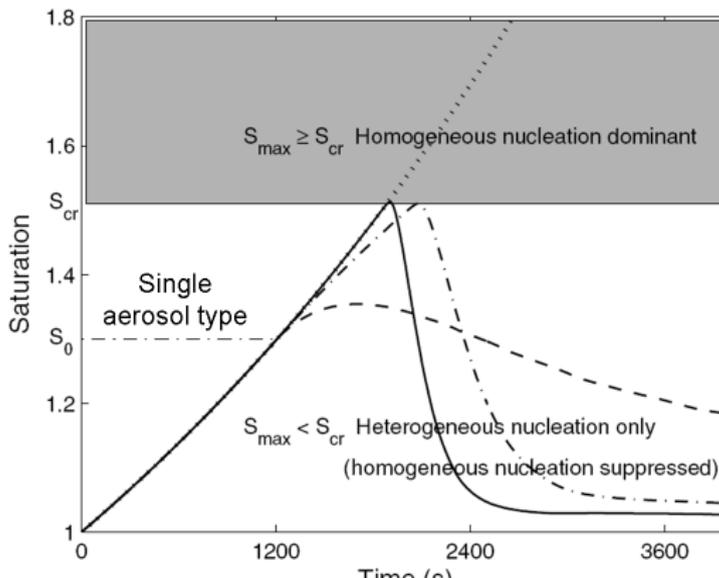
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Whereas in warm clouds the droplet concentration was related directly to the CCN number, in ice clouds the relationship between IN and ice particle number concentration is more complicated and nonlinear, falling into two distinct regimes, illustrated schematically in Fig 66.



The final number of ice crystals will be dependent on the both the IN concentration of the air and

The final number of ice crystals will be dependent on the both the IN concentration of the air and the updraught velocity of the air parcel. As in warm clouds, parcel models are used to study this, as in this example of Ren and Mackenzie (2005) in Fig.67. **In a weather or climate model, what are the issues involved in representing this relationship?**



The following points should be noted about the parcel model results in Fig.67:

- The **overshoot** past  $S_{cr}$  for homogeneous nucleation determines the **number** of ice crystal nucleated.

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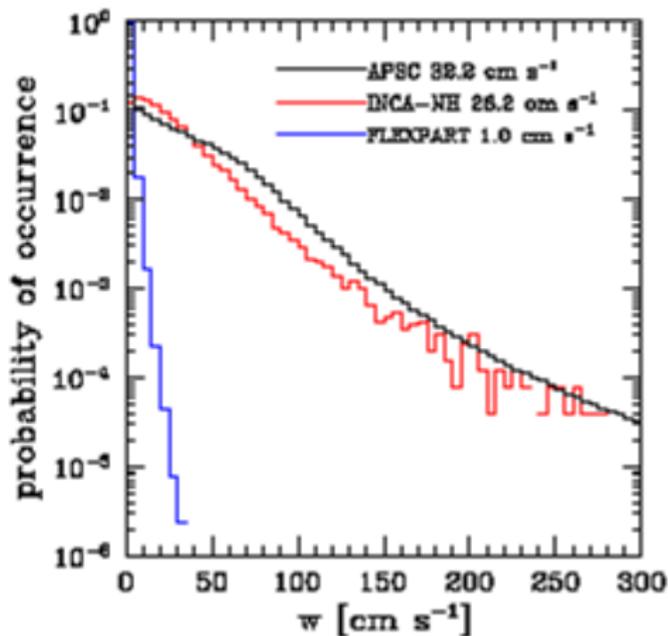
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Another example of this work is Gierens (2003).

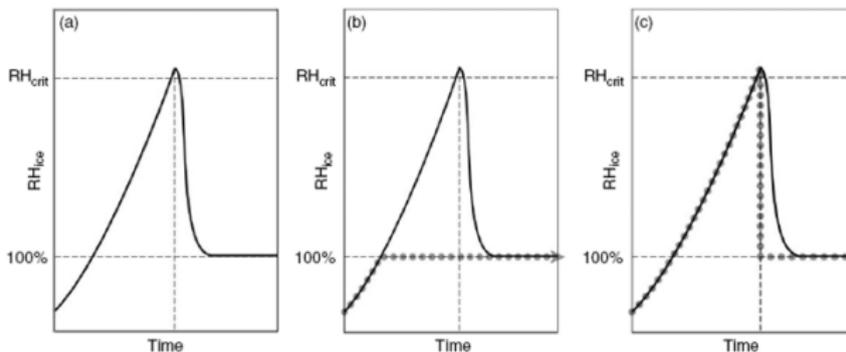
To pick up on the point concerning the model vertical velocity, Fig. 68 gives typical PDFs observed and also modelled using a global model. It is seen that the coarse resolution of the model truncates the spectra and does not resolve the highest velocities occurring on small spatial scales. Some success at reproducing the statistics of the vertical velocity spectrum is achieved with a simple parametrization.



**Figure:** Results from Lohmann (citation needed) showing PDF of vertical velocity from (blue) a global model (red) aircraft observations and (black) model results

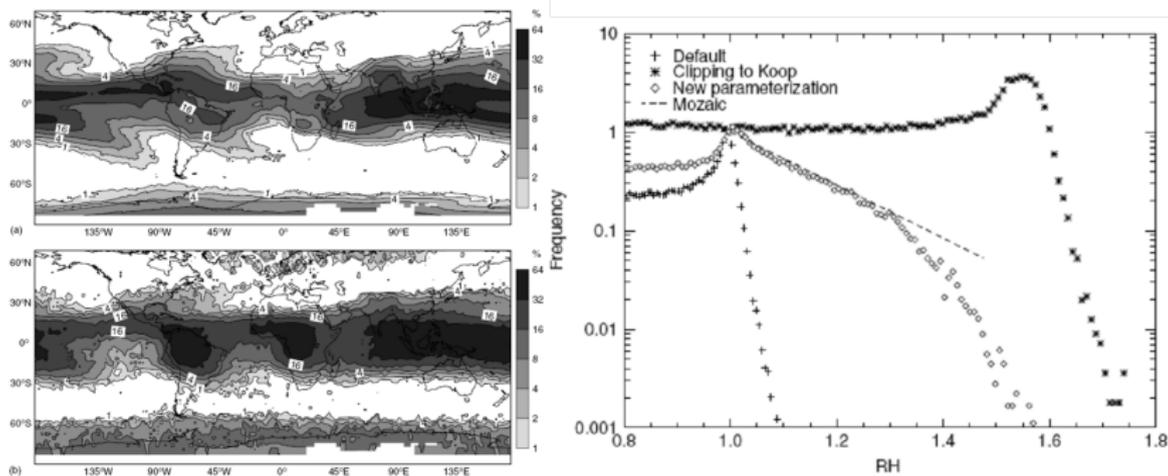
Some of my own work (Tompkins et al., 2007) has attempted to implement a modified cloud scheme into a weather forecast model that takes the zero-order effect of the elevated homogeneous nucleation threshold into account.

The schematic in 69 shows the  $RH$  evolution of a air parcel undergoing homogeneous nucleation and model assumptions the dotted line add shows the assumptions made in many models (middle panel) and the new scheme (right). The new scheme allows the clear air to be supersaturated, but assumes that once ice crystals are nucleated they grow instantaneously by diffusion to return the vapour to ice saturation within a model timestep. The **zero order** effect of the ice nucleation threshold and **hysteresis** behaviour is captured.



**Figure:** Schematic from Tompkins et al. (2007) showing (left) the  $RH$

Such a simple scheme is able to reproduce the distribution of frequency of occurrence of supersaturation from satellite retrievals and the PDF of  $RH$  from aircraft well (Fig. 70).



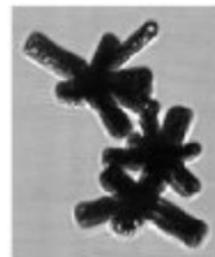
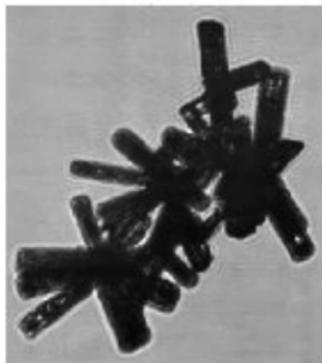
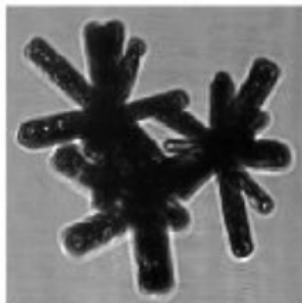
**Figure:** Results from Tompkins et al. (2007) showing (left) freq of occurrence of supersaturation compared to retrievals of Spichtinger et al. (2003) and (right) PDF of  $RH$  compared to Mozaic data of Gierens et al. (1999)

Errors in the map reflect errors in the convection occurrence in this version of the model known from other sources. We note (1) that model error sources may be remote and are not easy to track down (cancellation of errors) and (2) parametrization complexity does not need to exceed that necessary to represent the zero-order effect of the process, your knowledge of the process, or the complexity of the models' other components!

## Aggregation

The ice particles that form from diffusional growth from an ice crystal are called **pristine**. However pristine ice particles can clump together to form **snowflakes** in a process known as **aggregation**.

The efficiency of this process increases as the temperature exceeds  $-5^{\circ}\text{C}$ , when ice surfaces becomes *sticky*. There is also a secondary peak between  $-10^{\circ}\text{C}$  and  $-16^{\circ}\text{C}$  when dendrite arms get entangled. Figure shows an example of 71 of aggregates of bullet rosettes.

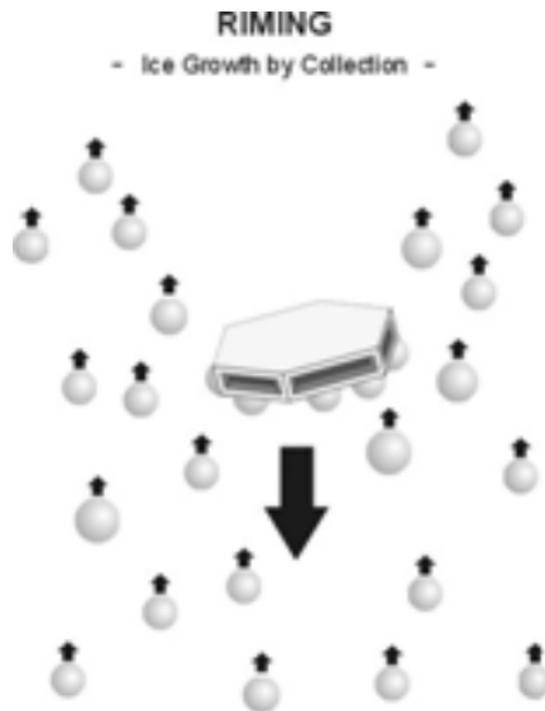


500 microns

## Riming to form graupel

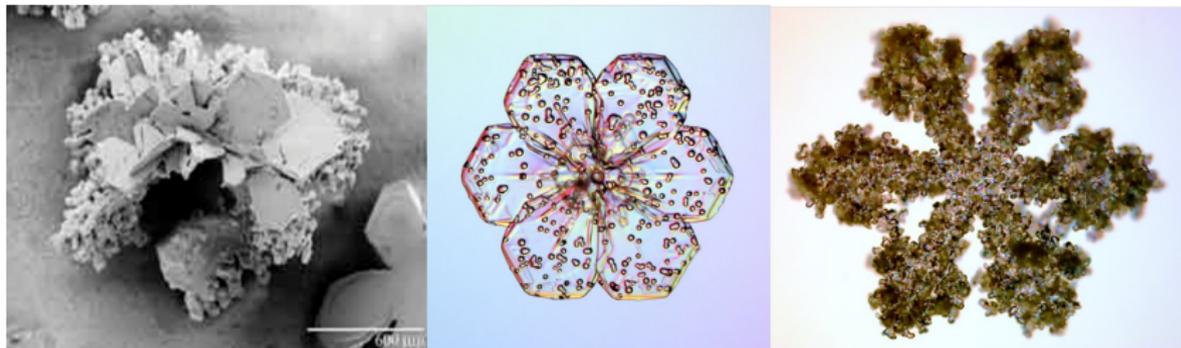
If vapour exceeds the water saturation mixing ratio (in strong updrafts) then water can condense on ice crystals, and then subsequently freeze to form **graupel**, which are round ice crystals of higher densities.

Graupel and Hail are also formed by aggregating liquid drops in mixed phase clouds (see schematic in Fig. 72).



**Figure:** Schematic of riming process (source unknown)

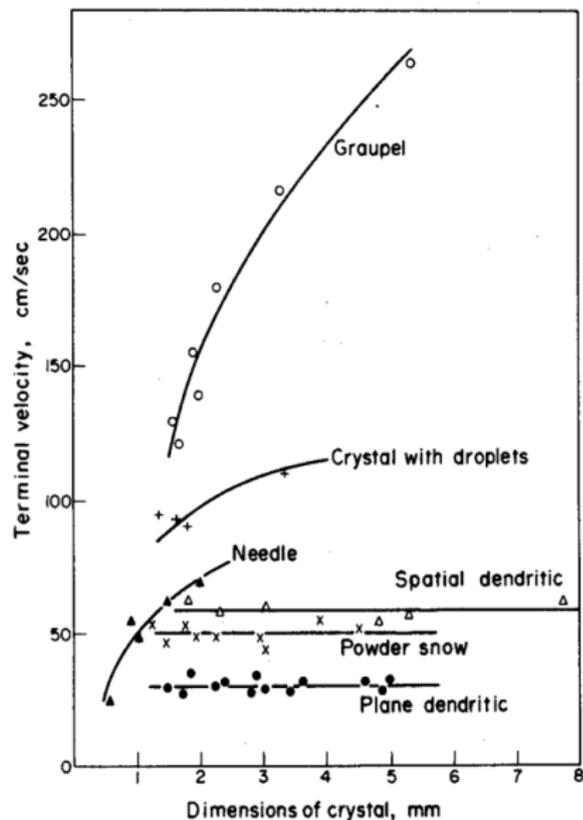
If the latent heat of condensation and fusion keeps temperature close to 273K then high density **hail** particles can form, since the liquid water “*spreads out*” before freezing. Hail is dense and thus has high terminal velocities (up to 40 m/s) implying that it only forms in convection with strong updraught able to support the particle long enough for growth



**Figure:** Photos of lightly and heavily rimed ice (source <http://www.its.caltech.edu/>)

The density and size of ice particles determines their respective fallspeeds and its dependence on particle radius (Fig. 74).

There are implications for numerical models that may become clear examining radar animations of cloud systems (see link). **can you think what these might be?**



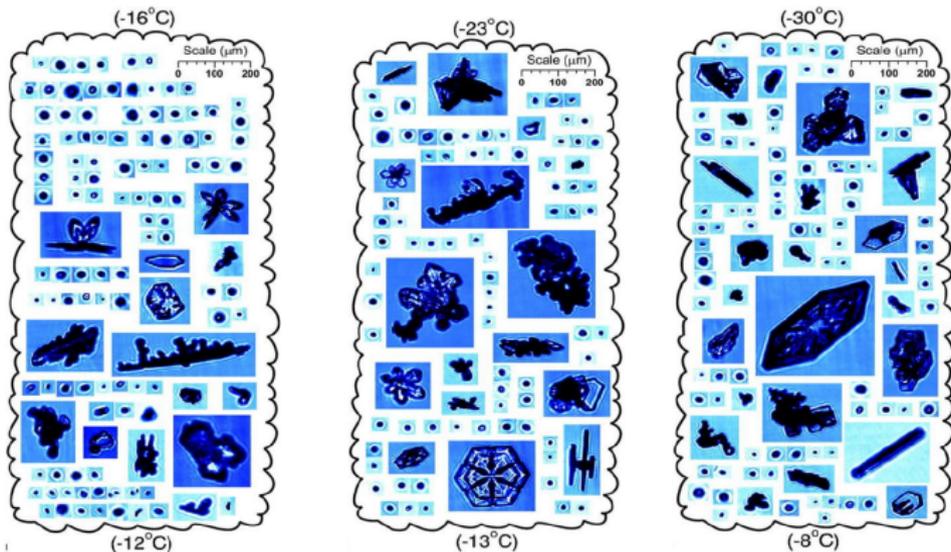
## Ice multiplication

In recent years it has become increasingly evident that concentrations of ice crystals in “real” clouds are not always represented by the concentrations of IN measured or expected to be activated in such environments. In particular, it has been found that at temperatures warmer than  $-10^{\circ}\text{C}$  the concentration of ice crystals can exceed the concentration of IN activated at cloud top temperature by as much as three or four orders of magnitude

There have been a number of hypotheses which have been advocated to explain such discrepancies. In particular, these hypotheses attempt to account for the unexpectedly high concentrations at warm temperatures. Some of the leading hypotheses are:

- 1** Fragmentation of large drops during freezing.
- 2** Mechanical fracture of fragile ice crystals (i.e., dendrites and needles) caused by collision of these crystals with graupel and other ice particles.
- 3** Splinter formation during riming of ice crystals.
- 4** Artificial measurement artifact of ice crystals shattering on impact with aircraft measurement device inlet.

Aircraft observations of ice in mid-level clouds observed in aircraft campaigns by Fleishauer et al. (2002) are shown in Fig. 75. Although many models make simple assumptions such that ice particle size reduces with temperature, it is seen that this is not necessarily valid in clouds with strong mixing. The small particles may only appear circular due to instrument resolution limitations.



## Raindrop size distributions

In many parts of the globe frozen precipitate melt before they reach the surface and thus precipitation reaches the surface in the form of rain. There is a natural limit to the raindrop size due to droplet breakup, and it is usually very rare to measure raindrops with radii larger than 3mm - with 2mm a more common limit.

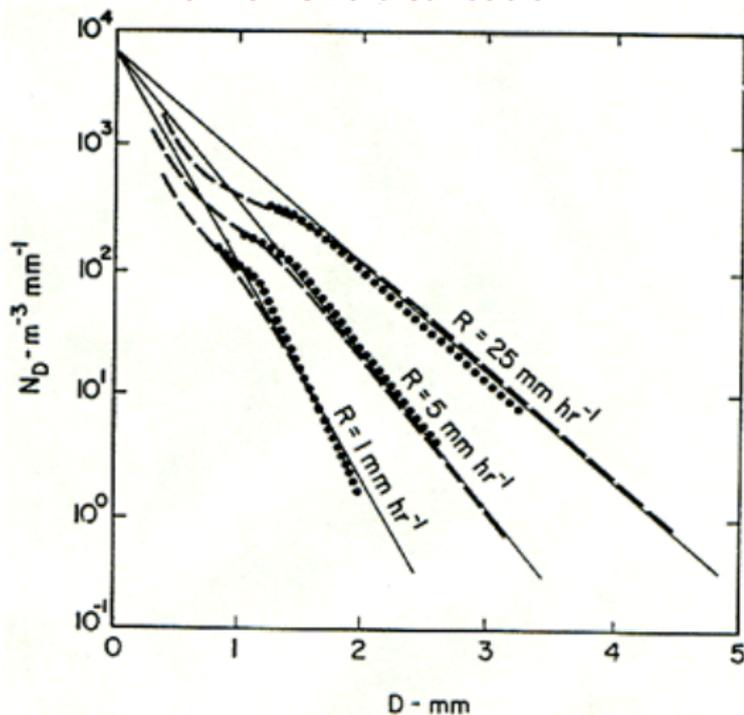
Q: If you stand underneath a tree during a shower, you may get much larger drops fall on you. Why?

The reduction in occurrence with droplet size implies that raindrop radii (or diameters) tend to follow an inverse exponential distribution, known as the Marshall-Palmer distribution (Marshall and Palmer, 1948), after the first authors to suggest the relationship from observations

$$N(D) = N_0 e^{-\Lambda D}, \quad (125)$$

where  $N(D)dD$  is the number of drops per unit volume with diameters between  $D$  and  $D + \Delta D$

Q: what properties do you notice from the rainfall size distribution?

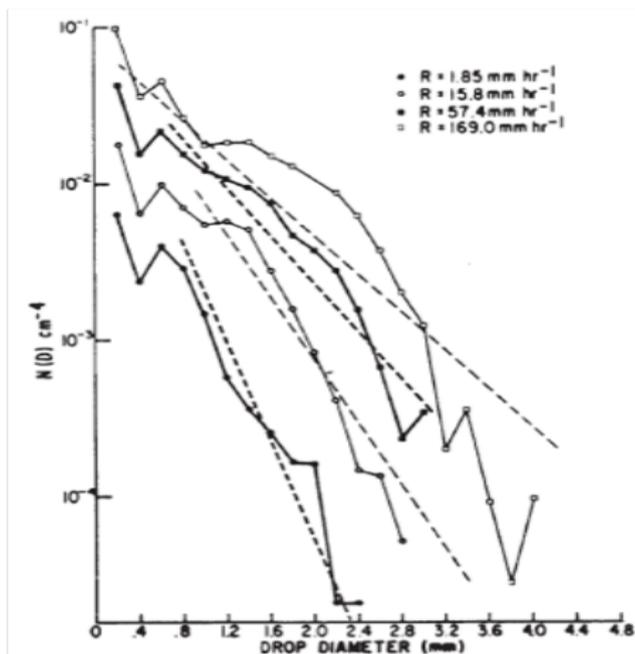


The slope factor  $\Lambda$  depends on rainfall rate  $R$  and is given by

$$-\Lambda(R) = 41R^{-0.21}, \quad (126)$$

where the units of  $R$  and  $\Lambda$  are  $\text{mm hour}^{-1}$  and  $\text{cm}^{-1}$ , respectively. It was also found that  $N_0$  was independent of rainfall rate, taking a value of  $N_0 = 0.08 \text{ cm}^{-4}$ .

Of course, these relationships are empirical and valid over a large-number of observations. Exact relationships will vary with location and time and departures from the MP distribution will always occur (Fig. 77).



## Snowflake size distributions

Most of the precipitation that reaches the ground does so as snowflakes and not pristine ice crystals.

**Q: What conditions are most likely required for pristine ice precipitation?.**

Since snowflakes are irregular aggregates of crystals there is no simple way to measure their dimensions, thus they are usually measured in terms of their mass or, equivalently, the diameter of the water drop that would form from their melting.

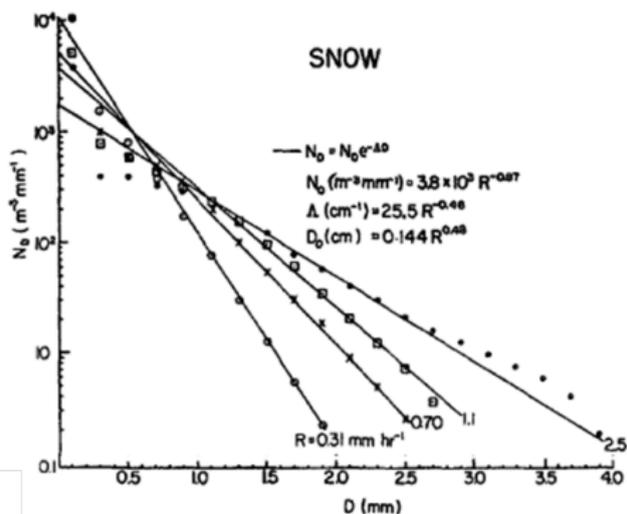
Gunn and Marshall (1958) found that the exponential approximation still fits the snowflake diameter well with the following parameters:

$$-\Lambda(R) = 25.5R^{-0.48}, \quad (127)$$

and

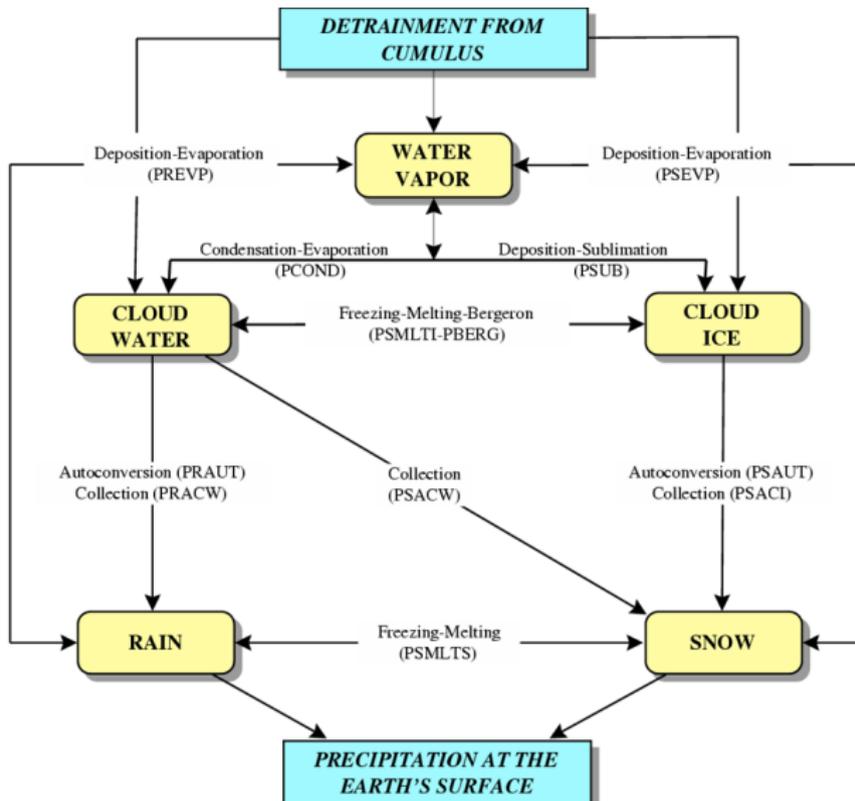
$$N_0 = 3.8 \times 10^{-2} R^{-0.87} \quad (128)$$

where the units of  $R$  and  $\Lambda$  are  $\text{mm hour}^{-1}$  and  $\text{cm}^{-1}$  (Fig 78).



**Figure:** Snow flake size distributions (from Gunn and Marshall, 1958)

The final figure (Fig. 79) shows an intermediate complexity bulk scheme in use in numerical models.



## Reminder Questions

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- explain at the board what it means for an aqueous aerosol solution droplet to become “activated”.
- What are the main mechanisms of ice nucleation that are relevant for ice cloud formation?
- Explain why you can sometimes see your breath in winter.

- How is a cloud called that consists only of ice, and one which consists of both liquid and ice? In roughly which temperature regimes does each cloud type exist? In the mixed phase explain what the Bergeron-Findeison effect is for ice growth

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- Does the ice crystal radius increase or decrease with ice nuclei (IN) number? What is a common source of IN?

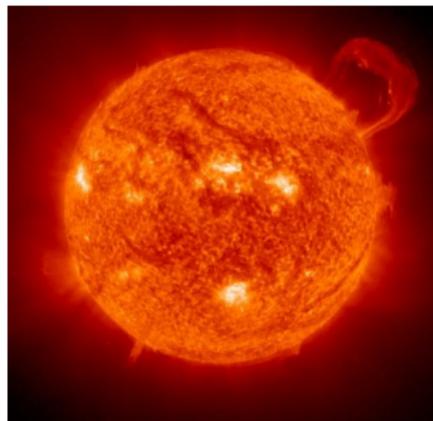
- What are the two main mechanisms by which activated aerosols grow into raindrops, and under which droplet size radius do they operate and why? We made a number of approximations in our mathematical descriptions of the two processes, can you name some?

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- What factor directly determines the number of ice crystals nucleated by each homogeneous nucleation, and what is this dependent on in turn?
- Explain what the criteria are for absolute convective stability and instability, and also conditional instability.

## Why study radiative processes?

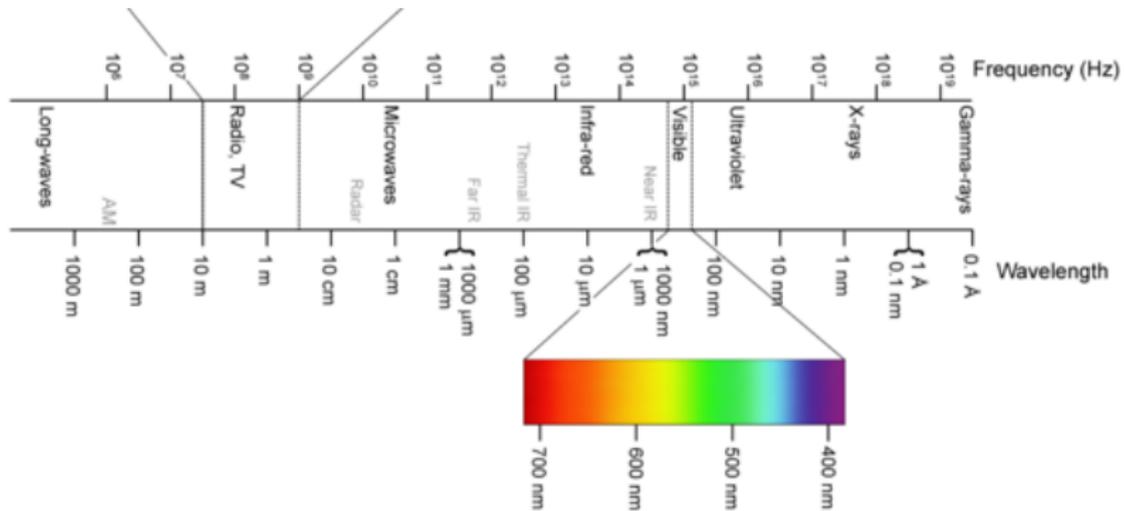
The sun (Fig. 80) is the fundamental energy source of the planet, and the uneven adsorption of solar radiation leads to the large-scale weather patterns that redistribute heat. Perturbations to radiative processes are responsible for climate variability through the greenhouse effect, volcanic eruptions and solar variability. Understanding radiative processes is also important for remote sensing of the atmosphere.



**Figure:** The sun! (<http://z.about.com>)

## Electro-magnetic spectrum

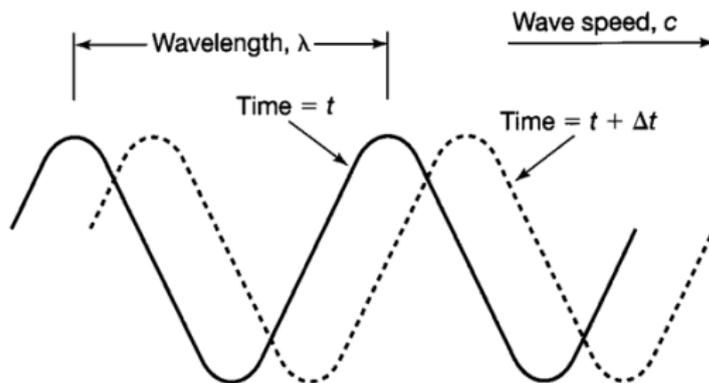
The electromagnetic (EM) spectrum is the range of all possible electromagnetic radiation frequencies. The "electromagnetic spectrum" of an object is the characteristic distribution of electromagnetic radiation from that particular object.



## Electro-magnetic wave

Electro-magnetic waves have a speed of  $c = 3 \times 10^8$  m/s, (Fig. 82) with speed, frequency  $\nu$  and wavelength  $\lambda$  related by

$$c = \nu\lambda \quad (129)$$



**Figure:** Propagating electromagnetic wave.

The Earth with radius<sup>9</sup>  $r_e$  orbits the sun ( $r_s$ ) at a radius of  $r_d$  and intercepts a tiny fraction of the emitted energy (see sketch). We assume that the sun emits as a **black body**.

---

<sup>9</sup>In the radiation section  $r$  is used for radius, not to be confused with mixing ratio! Other duplications of notation will also occur. 

## Black Body

A black body is an object that absorbs all electromagnetic radiation that falls on it. No electromagnetic radiation passes through it and none is reflected.

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## Irradiance and Radiant Emittance $E$

**Irradiance** and **radiant emittance** are radiometry terms for the power of electromagnetic radiation arriving or leaving a flat-plate surface, per unit area, integrated over the **hemisphere** above the plate (sketch).

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The total energy per unit time is called the **radiant flux** and has units of  $\text{J s}^{-1}$  or  $\text{W}$ . As the irradiance and radiant emittance are measured per unit area they have units  $\text{W m}^{-2}$ . There is nothing in the definition of irradiance that tells us from which direction the radiation arrives or is emitted. We therefore introduce the **solid angle**.

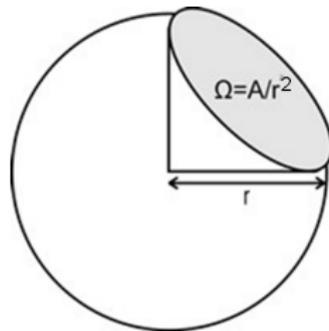
## Solid Angle $\Omega$

The solid angle,  $\Omega$  is the angle in three-dimensional space that an object subtends at a point. It is a measure of how big that object appears to an observer looking from that point. The solid angle is proportional to the surface area,  $A$ , of a projection of that object onto a sphere centered at that point, and inversely proportional to the square of the sphere's radius,  $r$  (Fig. 83).

We define the solid angle

$$\Omega = \frac{A}{r^2} \quad (130)$$

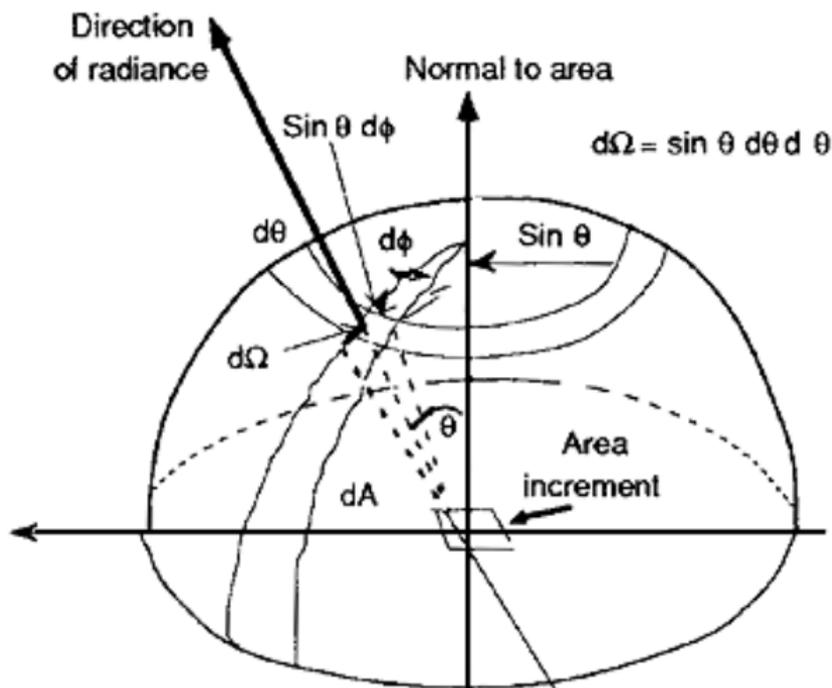
and it has units of **steradians** or **ster**.



**Figure:** Definition of the solid angle.

An important special case for radiation is the integration over a hemisphere of unit radius in spherical coordinates (Fig. 84).

$$\int_{\text{hemisphere}} d\Omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \sin\theta d\theta d\phi = 2\pi \text{ steradians} \quad (131)$$



## Radiance

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The radiance tells us the radiant flux density per unit solid angle coming from a direction  $(\theta, \phi)$ . For a surface area of  $\Delta A_{\text{norm}}$  **normal** to the direction  $(\theta, \phi)$ , the radiant flux received from the solid angle  $\Delta\Omega$  will be  $L(\theta, \phi)\Delta\Omega\Delta A_{\text{norm}}$  (**draw sketch**).

$L(\theta, \phi)$  is the **radiance** with units  $\text{W m}^{-2} \text{ster}^{-1}$

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Thus in general the contribution to the radiant flux incident on a horizontal surface is given by:  $L(\theta, \phi) \Delta\Omega \Delta A \cos\theta$ .

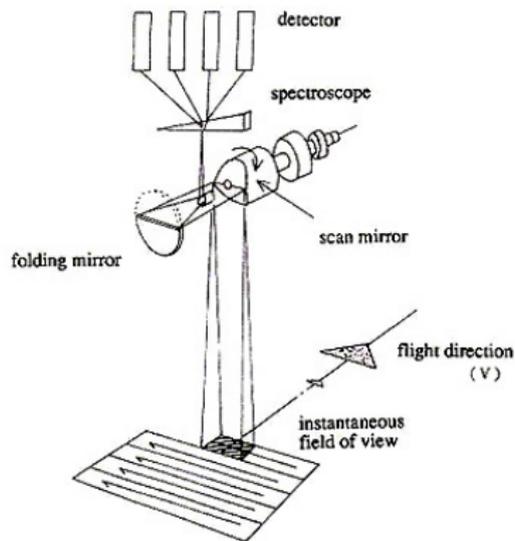
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Thus in general the contribution to the radiant flux incident on a horizontal surface is given by:  $L(\theta, \phi)\Delta\Omega\Delta A\cos\theta$ . If we take  $\Delta A = 1$  for a unit area, then the radiant flux per unit area from solid angle (i.e. the contribution to the **irradiance** from that solid angle) is  $L(\theta, \phi)\cos\theta\Delta\Omega$  with units  $\text{W m}^{-2}$ .

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With remote sensing we need a device capable of measuring radiation from different directions with very small solid angles to achieve the best resolution possible (Fig. 85).



**Figure:** Schematic of why small solid angles measurements are required for remote sensing (source NASA website).

## Irradiance $E$ Definition

We are now able to return to the concept of **irradiance**, which is the total radiance integrated over all solid angles over a hemisphere.

Thus

$$E = \int_{\text{hemisphere}} L \cos\theta d\Omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} L \cos\theta \sin\theta d\theta d\phi \quad (132)$$

## Isotropic radiation

An important special case is **isotropic radiation**, namely that the radiative flux has the same intensity regardless of the direction, and thus  $L$  is independent of direction. **Q: can you think of some examples?**

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- Emission from a black body.

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An important special case is **isotropic radiation**, namely that the radiative flux has the same intensity regardless of the direction, and thus  $L$  is independent of direction. **Q: can you think of some examples?** Examples include:

- Emission from a black body.
- Solar radiation beneath a cloud of moderate thickness due to multiple scattering of photons.
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In this case we may write

$$E = L \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta d\theta d\phi = \pi L \quad (133)$$

**Exercise: show!**

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## Planck Function

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where  $c_1$  and  $c_2$  are constants.

## Planck Function

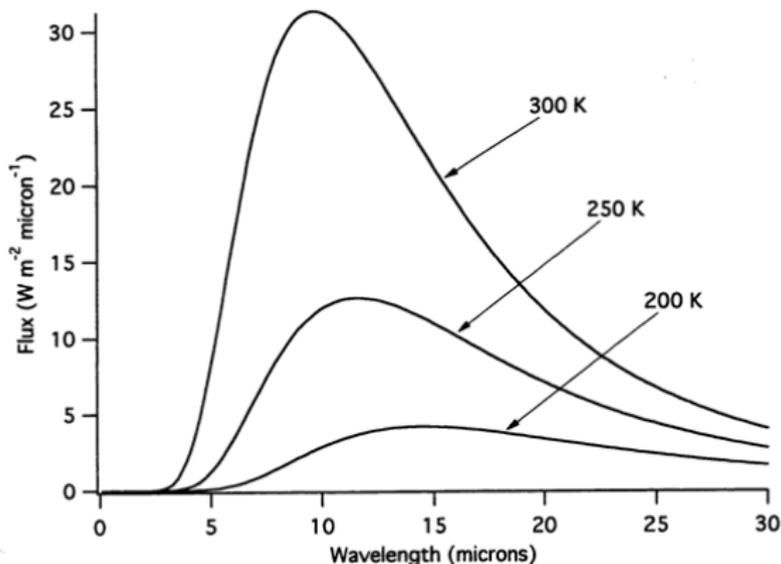
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where  $c_1$  and  $c_2$  are constants. The emitted power per unit area of emitting surface, per unit solid angle, and per unit wavelength.

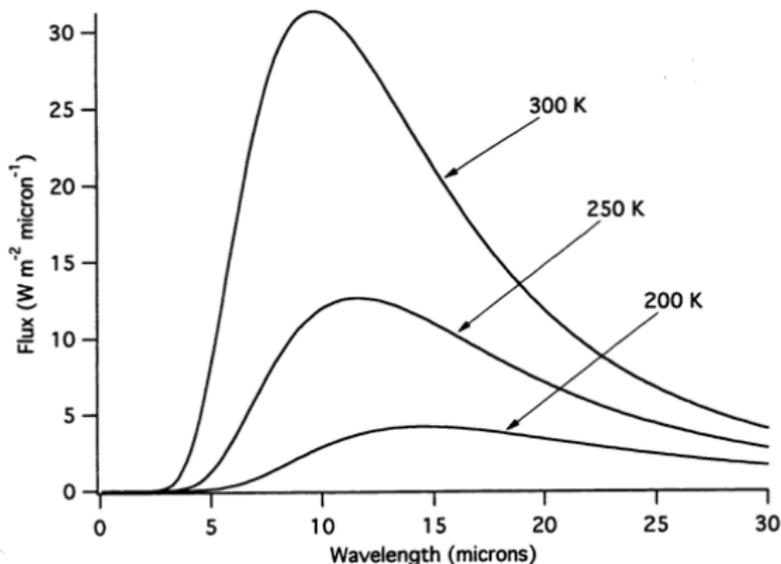
At room temperature, black bodies emit mostly infrared light ( $\lambda > 4\mu\text{m}$ ), but as the temperature increases past a few hundred degrees Celsius, black bodies start to emit visible wavelengths, from red, through orange, yellow, and white before ending up at blue, beyond which the emission includes increasing amounts of ultraviolet.

Examples of the Planck function for temperatures between 200 to 300 K are given in Fig. 86.



**Figure:** Black body emission at three tropospheric temperatures.

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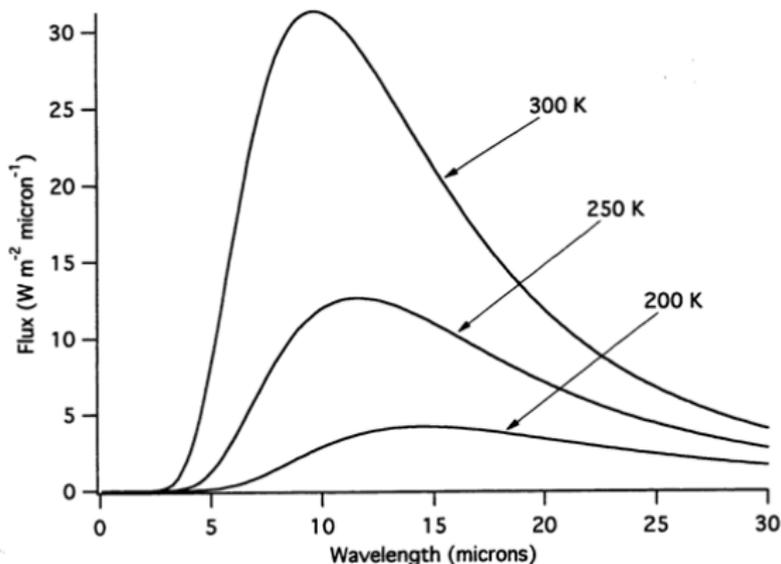


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## Wein's Displacement Law

Note how the peak in Fig. 86 moves to shorter wavelengths as the temperature increases.

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**Figure:** Black body emission at three tropospheric temperatures.

## Wein's Displacement Law

Note how the peak in Fig. 86 moves to shorter wavelengths as the temperature increases. The relationship describing the wavelength at which emission is at a maximum (i.e. where  $dL_\lambda/d\lambda = 0$ ) is

$$\lambda_{\max} = \frac{2897 \mu\text{K}}{T} \quad (135)$$

Integrating (134) across all wavelengths gives the total emitted energy **per unit solid angle**, in other words the radiance  $L$ . We will not enter the details of the mathematics here, but simply state the result that

$$L \equiv \int_0^{\infty} L_{\lambda} d\lambda = \frac{\sigma T^4}{\pi} \quad (136)$$

where  $\sigma$  is the Stefan Boltzmann constant ( $= 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ ).

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We now recall that the integral of  $L$  over the hemisphere gives us the Radiant Emittance  $E$ , which (133) tells us in the case of isotropic blackbody radiation is equal to  $\pi L$ . Thus combining (134) and (133) we get

$$E = \sigma T^4 \quad (137)$$

This is known as the **Stephan-Boltzmann Law**.

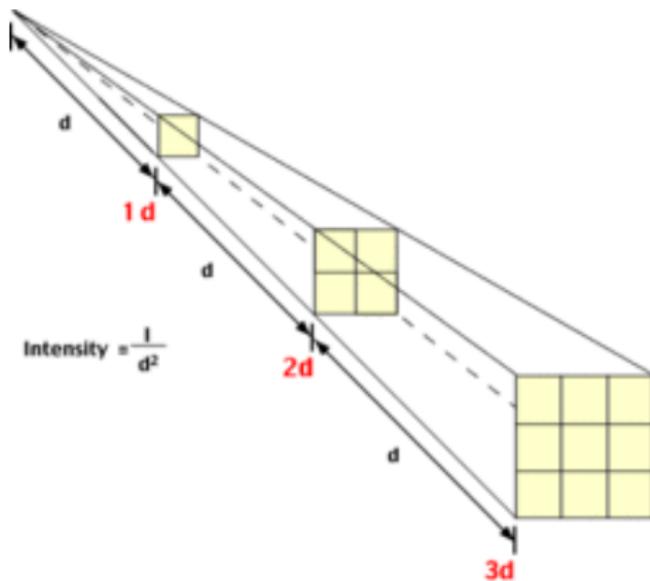
## Energy balance models of the atmosphere

We are now in a position to construct a simple energy balance model of the earth-atmosphere system. If the sun is considered to be a black body (a good approximation) then its emittance is given by  $\sigma T_{\text{sun}}^4$  where  $T_{\text{sun}}$  is the surface temperature of the sun.

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The energy of the sun is spread over the surface of a sphere of area  $4\pi r_d^2$  at a distance of  $r_d$ , known as the inverse square law (Fig. 87).



**Figure:** Inverse square law, radiation spread over increasing

The **irradiance**  $E$  ( $\text{W m}^{-2}$ ) at a distance  $r_d$  is thus

$$E = \sigma T_{\text{sun}}^4 \left( \frac{r_s}{r_d} \right)^2. \quad (138)$$

If  $r_d$  corresponds to the distance of separation between the earth and the sun, then we define  $S_0 \equiv E$ , called the **solar constant**.

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### Solar Constant $S_0$

$S_0$ , the average irradiance at the top of the earth's atmosphere, is known as the **Solar Constant**. Measurements from satellites show that  $S_0 \simeq 1370 \text{ W m}^{-2}$ . The value varies by  $\approx 1 \text{ W m}^{-2}$  over a solar cycle that has an average length of 11 years.

Knowing the solar constant we can invert (138) to get the surface temperature of the sun:

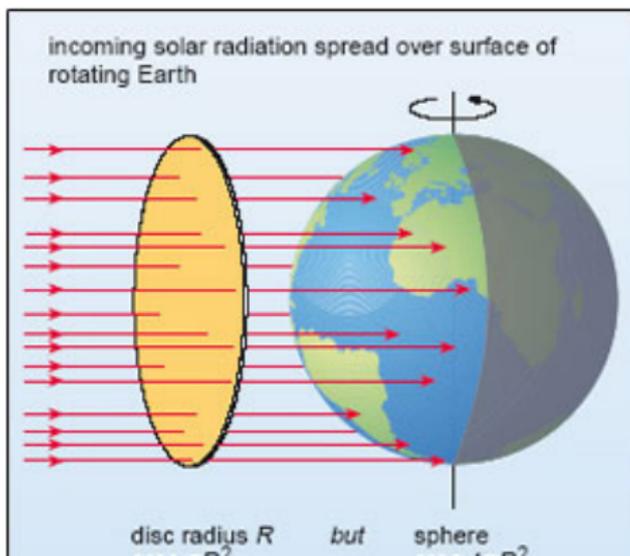
$$T_{\text{sun}} = \sqrt[4]{\left(\frac{r_d}{r_s}\right)^2 \frac{S_0}{\sigma}} \quad (139)$$

giving  $T_{\text{sun}}$  as 5800K.

The energy intercepted by the earth is (see Fig.88)  $S_0\pi r_e^2$ , so that  $E_e$ , the average incident solar radiation on the earth's surface, which has a surface area of  $4\pi r_e^2$ , is

$$E_e = \frac{S_0\pi r_e^2}{4\pi r_e^2} = \frac{S_0}{4} \quad (140)$$

which has a value of  $340 \text{ W m}^{-2}$ .



## Planetary Albedo

A proportion of this radiation is scattered back to space by clouds, atmospheric gases and the planet's surface. We thus introduce the parameter  $\alpha_p$  as the **planetary albedo**. Which is defined as

$$\alpha_p = \frac{\text{Reflected solar irradiance (at top of atmosphere)}}{\text{Incident solar irradiance}} \quad (141)$$

Observations from Satellites put the earth albedo at  $\alpha_p \simeq 0.3$ .

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Observations from Satellites put the earth albedo at  $\alpha_p \simeq 0.3$ . Thus the average irradiance absorbed is  $\frac{S_0}{4}(1 - \alpha_p)$  which is  $240 \text{ W m}^{-2}$ .

## Effective Emitting temperature of the Earth

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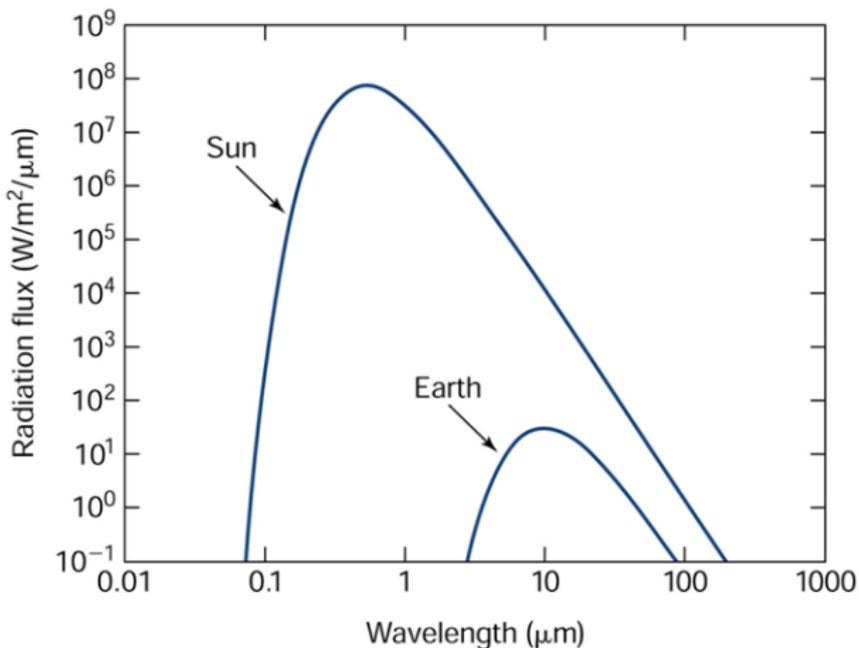
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Thus

$$T_e = \sqrt[4]{\frac{S_0}{4\sigma} (1 - \alpha_p)} = 255\text{K} \quad (143)$$

Thus we have two contrasting temperature of the sun (5800K) and the earth (255K). This is important since the peak wavelengths ( $\lambda$ ) at which energy is emitted is a function of temperature.

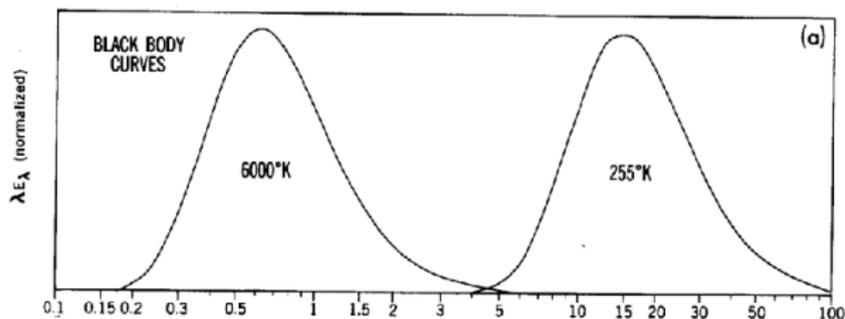
The Planck function  $L_\lambda$  is shown graphically for the sun and earth effective emitting temperatures in Fig. 89.



**Figure:** Radiative flux per micron for sun and earth's emitted temperatures (source unknown)

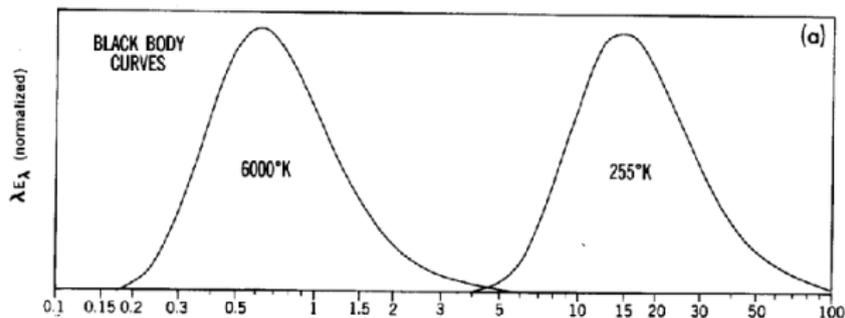
We now plot  $\lambda L_\lambda$  to give us the black body emitted radiation and reduce the solar curve by a factor of  $\frac{r_s}{r_d}(1 - \alpha_p)$  to get the normalized radiation at the earth, depicted in Fig. 90.

It should be pointed out the solar radiation energy is **not** restricted to the visible ( $0.4$  to  $0.7 \mu\text{m}$ ), rather 35% of the solar energy received at the earth is in this band, with 50% in the **near-infrared** and 15% in the **ultra-violet**.



**Figure:** Blackbody spectra after normalisation at sun/earth temperatures. (x-axis in  $\mu\text{m}$ ).

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It is also clear that the result of the different emitting temperatures is that the radiation from the sun and earth are in **distinct wavelength regions** with little overlap.

We found that  $T_e = 255$  K and yet surface measurements indicate an average global temperature of roughly 288K. Q: Why are these so different?

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### A simple model for the greenhouse effect

To illustrate the **Greenhouse Effect** we will now construct a simple radiative balance model for the earth surface with an overlying atmosphere which will be considered a single radiatively active **homogeneous slab**.

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We make the following simplifying assumptions:

- The earth surface is a black body.
- The atmosphere does not absorb solar radiation.
- Atmosphere can be represented as a **grey body**.

## Grey Bodies and Kirchoff's Law

A grey body emittance is one which emits radiation according to

$$E_{\text{grey}} = \epsilon \sigma T^4, \quad (144)$$

where  $\epsilon$  is the fractional emittance (  $\epsilon \leq 1$  ).

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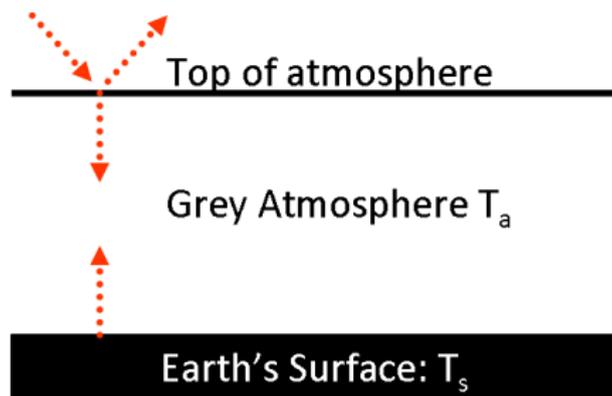
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**Kirchoff's Law** states that, if at a given wavelength we know how good an emitter a body is, then we also know how good an absorber it is. Kirchoff's Law specifies:

- Absorptance  $a = \epsilon$
- Transmittance  $\tau = 1 - \epsilon$

We can now write down the radiative energy balance of each component (according to the sketch in Fig. 91), assuming a state of **radiative equilibrium**.



**Figure:** Sketch of the greenhouse slab model (source: me!)

**Q:** Derive these components.

## Top of atmosphere

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$$\frac{S_0}{4}(1 - \alpha_p) = \sigma T_s^4(1 - \epsilon) + \epsilon\sigma T_a^4 \quad (145)$$

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## Atmosphere

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Q: Why is there a factor 2 on the RHS?

## Surface

$$\epsilon\sigma T_a^4 + \frac{S_0}{4}(1 - \alpha_p) = \sigma T_s^4 \quad (147)$$

Q: Why isn't there an  $\epsilon$  factor in the solar term?

We thus have 3 equations with 2 unknowns  $T_s$  and  $T_a$ , and can use any two to calculate these unknowns. For example, from (146) we have

$$T_a = \frac{T_s}{\sqrt[4]{2}}, \quad (148)$$

which if we substitute into (145) gives

$$T_s = \sqrt[4]{\frac{S_0(1 - \alpha_p)}{2\sigma(2 - \epsilon)}} \quad (149)$$

Exercise: Check this derivation and also try other equation combinations using (147)

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**Exercise: Check this derivation and also try other equation combinations using (147)**

If we assume that  $\epsilon = 0$  then we get the same result we achieved earlier, namely  $T_s = 255\text{K}$ , **Q: Why?**

A more reasonable value is  $\epsilon = 0.6$ , which results in  $T_s = 278\text{K}$  and  $T_a = 235\text{K}$ . Thus we can see that introducing an atmosphere that can absorb in the thermal IR **causes a surface warming**. This is known as the **greenhouse effect**. We will later consider what determines  $\epsilon$  and its variation in a continuous media.

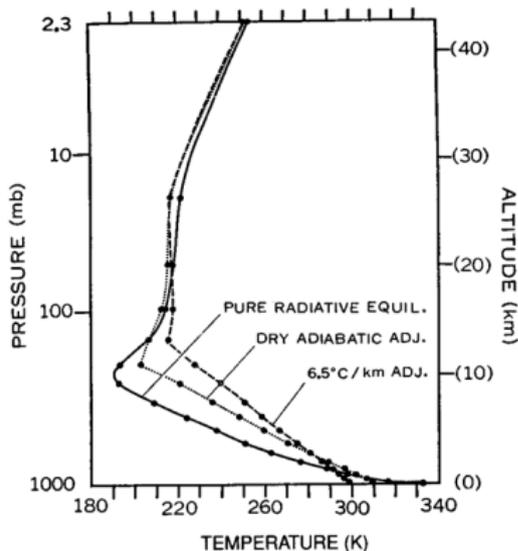
We could take this approach further and divide the atmosphere up into many vertical slabs, each with its own emittance according to the absorbing gases present in the layer. This would provide a **vertical atmospheric temperature profile** of the earth-atmosphere system in a state of **radiative equilibrium** (sketch).

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If one calculates the vertical temperature structure that would arise from radiative equilibrium it turns out that it is **absolutely unstable** to both dry and moist deep convection. Thus convection can (and does) occur to transport heat from near the earth's surface to higher in the atmosphere.

One of the earliest studies to examine this using a column model of the atmosphere was conducted by Manabe and Strickler (1964) (Fig. 92).

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**Figure:** Temperature profiles resulting from a pure radiative equilibrium, or a

Q: in a state of radiative equilibrium, what would be the net radiative heating everywhere in the atmosphere?

Q: in a state of radiative equilibrium, what would be the net radiative heating everywhere in the atmosphere? In a radiative equilibrium the net heating rate would be zero everywhere by definition. Instead in the **troposphere** there is a net cooling rate of  $O(1 \text{ K day}^{-1})$  which is balanced by convective heating processes.

## Sun-Earth Geometry

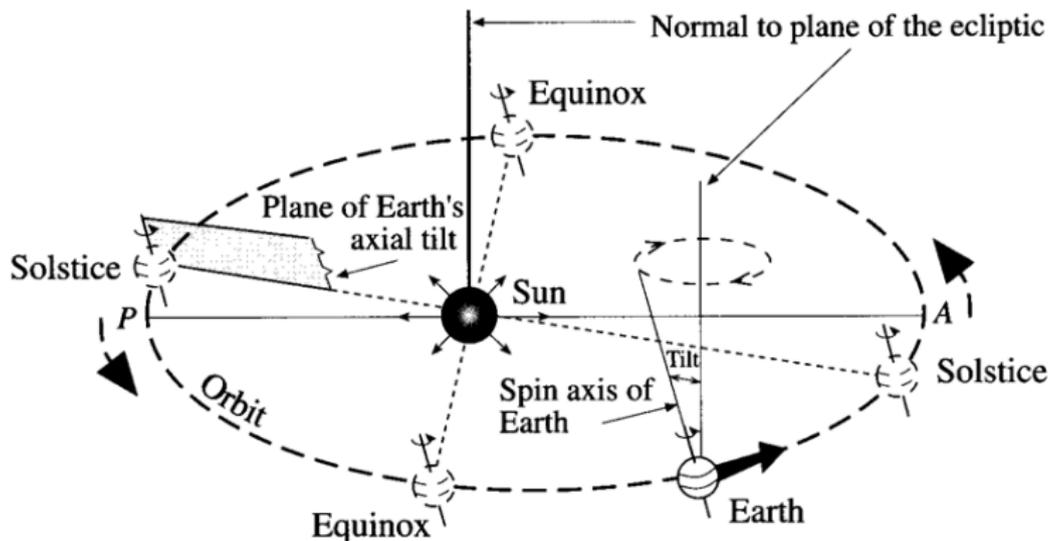
The sun-earth geometry is shown in Fig.93. Note the

- 1 Declination angle of the earth  $\vartheta$

## Sun-Earth Geometry

The sun-earth geometry is shown in Fig.93. Note the

- 1 Declination angle of the earth  $\vartheta$
- 2 Eccentricity of the earth's orbit



## Declination Angle $\vartheta$

- $\vartheta = +23.5^\circ$  : at the summer solstice June 21st
- $\vartheta = -23.5^\circ$  : at the winter solstice December 21st
- $\vartheta = 0^\circ$  : at the spring and Autumn Equinoxes

## Eccentricity of orbit

The Earth's orbit is slightly elliptical with an **eccentricity factor** of  $e = 0.0167$ . Since the average sun-earth separation is  $r_d = 149.6 \times 10^6 \text{ km}$ , we can calculate:

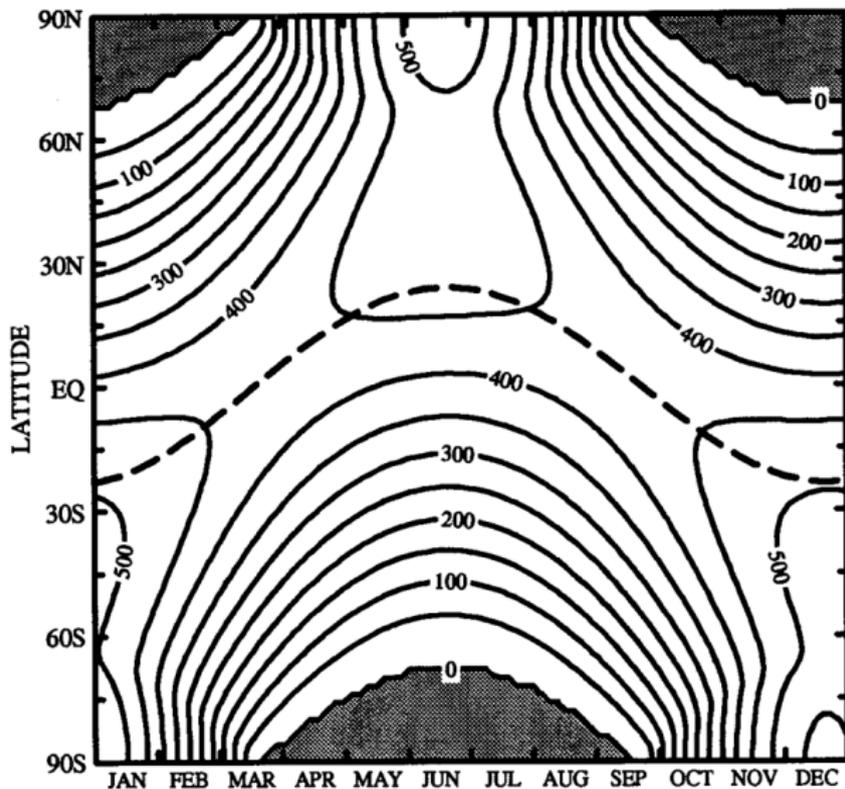
- The **Perihelion** (distance of closest approach) is  $r_d(1 - e) = 147.1 \times 10^6 \text{ km}$ .
- Thus the **Aphelion** (distance of furthest approach) is  $r_d(1 + e) = 152.1 \times 10^6 \text{ km}$ .

Presently, the Perihelion occurs on January 3rd while the Aphelion July 4th, but the closeness of these dates to the solstices is a present-day coincidence.

## Solar Zenith Angle

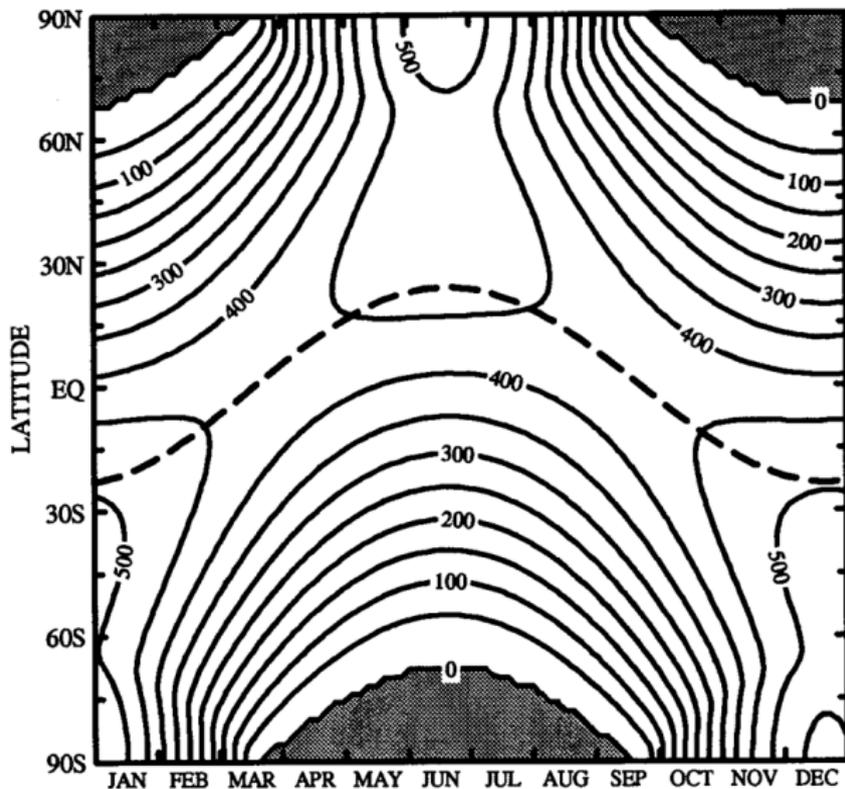
The solar zenith angle ( $\theta$ ) is a function of latitude, time of year and time of day. The **daily averaged** insolation is given in Fig. 94.

**Q: At midsummer, at which latitude do you think  $Q_{av}$  is greatest: Tropics, subtropics, midlatitudes or at the north pole?**



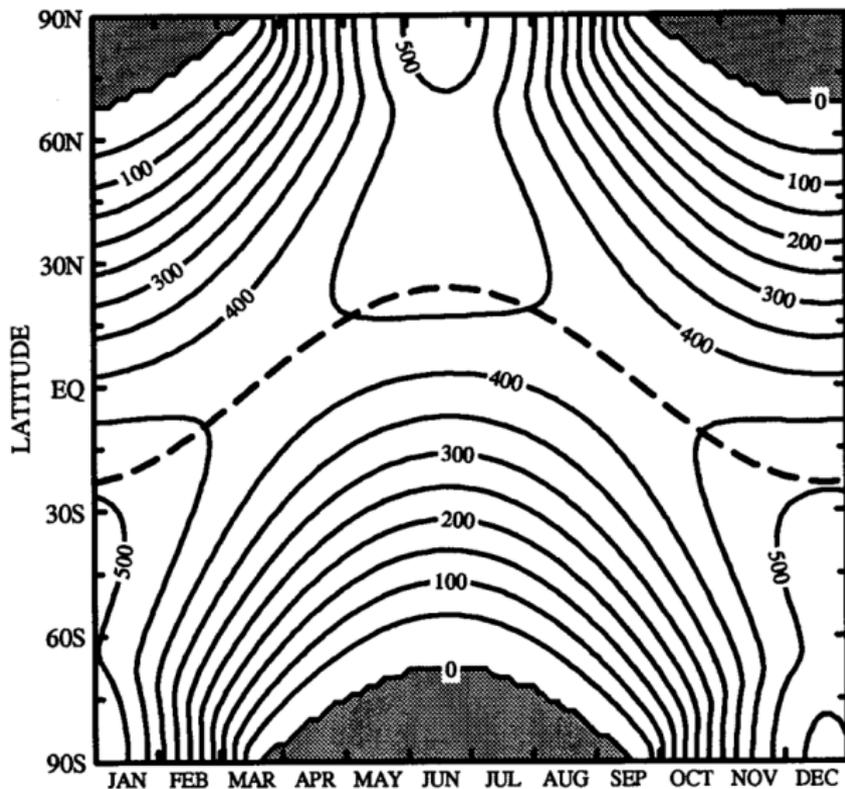
In midsummer  $Q_{av}$  is maximum at the north pole.

**Figure:** Day averaged insolation  $Q_{av}$  ( $\text{W m}^{-2}$ ) at the



In midsummer  $Q_{av}$  is maximum at the north pole.  $\theta$  ranges from  $0^\circ$  at a latitude of  $23.5^\circ\text{N}$  to  $66.5^\circ$  at the north pole. **Q: If the sun is overhead at  $23.5^\circ\text{N}$ , why is  $Q_{av}$  is maximum at the north pole?**

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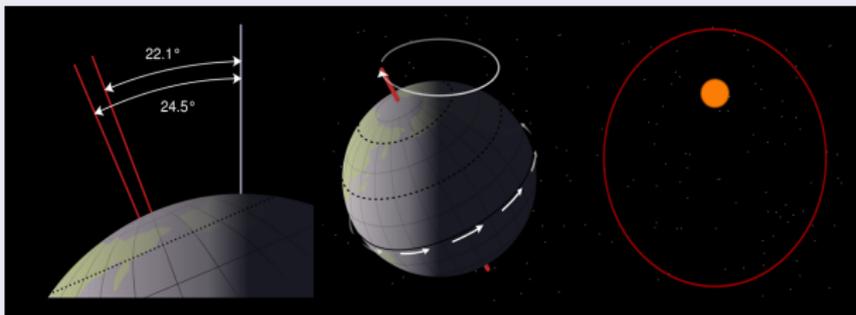


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**Figure:** Day averaged insolation  $Q_{av}$  ( $\text{W m}^{-2}$ ) at the

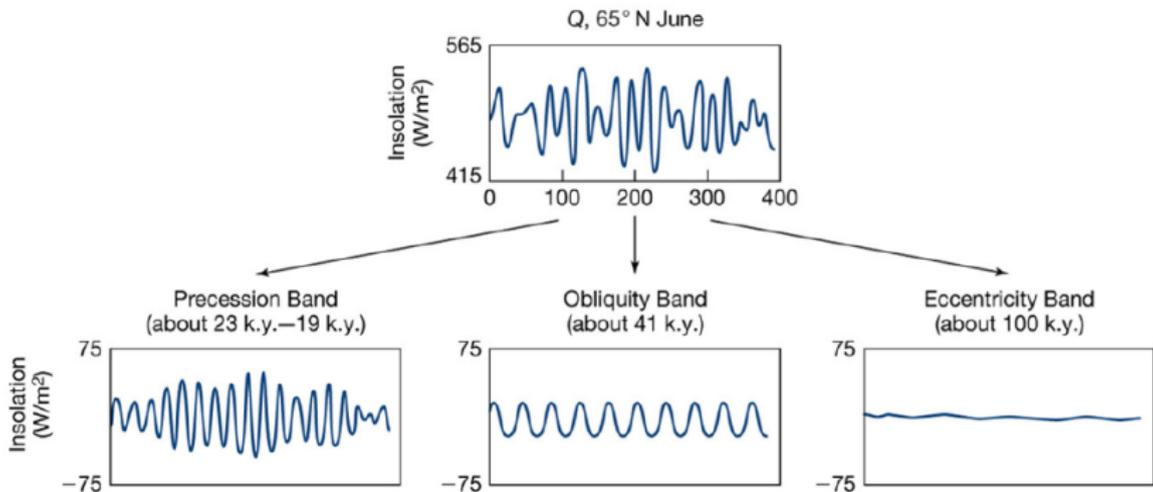
## Variations in Earth's orbit (Milankovitch cycles)

There are variations in the Earth's orbit parameters, that affect the eccentricity of the earth orbit, the Axial tilt (obliquity) and Precession (wobble).



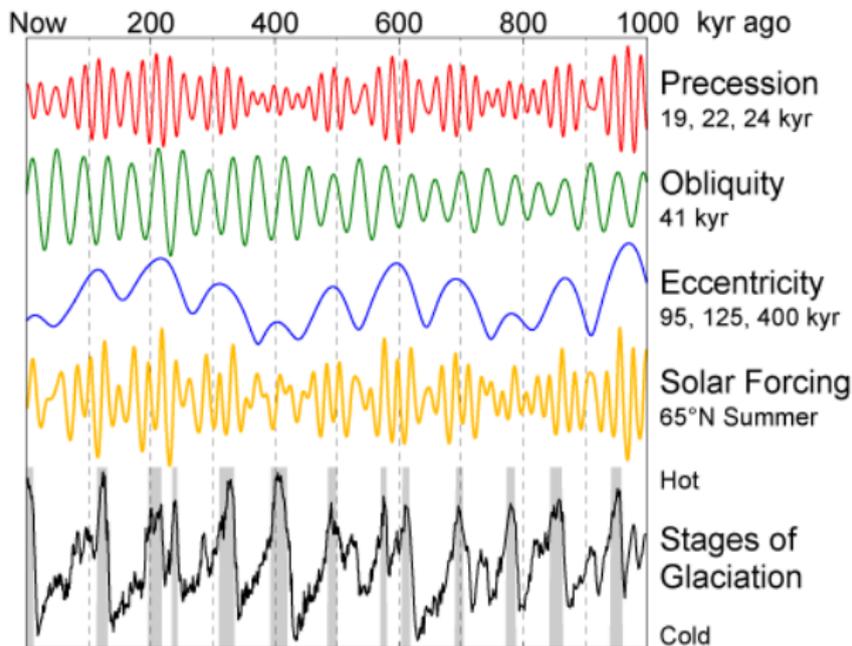
**Figure:** Schematic (Left) Axial tilt (obliquity), (middle) Precession and (right) eccentricity (source Wikipedia)

The variations in the solar insolation can strongly modulate climate (Fig. 96).



**Figure:** Solar flux variation as a result of earth orbit variations

However, these variations are not thought to be the causes of ice-ages beginning or ending, as the forcing variability occurs at a higher frequency than the major glaciations (Fig. 97),



**Figure:** Solar flux variations as a result of earth orbit variations and the

## Radiation interaction with a slab

We now consider the interaction of radiation with a layer of the atmosphere in more detail, considering the processes depicted in the sketch. We imagine a layer that is homogeneous in the infinite horizontal direction, and has a finite vertical depth of  $\Delta z$ .

The radiance  $L_\lambda$  which we recall is a function of  $(\theta, \phi)$ , can be divided into three distinct components (draw sketch):

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- 1** The direct solar component: the radiative flux that passes unimpeded through the slab direction  $(\theta, \phi)$ .

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- 1** The direct solar component: the radiative flux that passes unimpeded through the slab direction  $(\theta, \phi)$ .
- 2** The scattered component: the radiative flux that originates from a different solid angle to  $(\theta, \phi)$ , but undergoes scattering

## Radiation interaction with a slab

We now consider the interaction of radiation with a layer of the atmosphere in more detail, considering the processes depicted in the sketch. We imagine a layer that is homogeneous in the infinite horizontal direction, and has a finite vertical depth of  $\Delta z$ .

The radiance  $L_\lambda$  which we recall is a function of  $(\theta, \phi)$ , can be divided into three distinct components (draw sketch):

- 1** The direct solar component: the radiative flux that passes unimpeded through the slab direction  $(\theta, \phi)$ .
- 2** The scattered component: the radiative flux that originates from a different solid angle to  $(\theta, \phi)$ , but undergoes scattering within the slab to emerge in direction  $(\theta, \phi)$ .
- 3** Emission: Radiation that is emitted from the slab in direction  $(\theta, \phi)$ .

## Direct Radiation - Beer's Law

Beer's Law The law states that there is a logarithmic dependence between the transmission  $\tau$  of the a radiative flux through a substance and the product of the absorption coefficient of the substance,  $a$ , and the thickness of the material (i.e. the path length).

To derive this, we note that the finite reduction in the direct component by the slab  $dL_\lambda$  will be directly proportional to (**draw sketch**):

- $L_\lambda$ : The original intensity of the radiative beam

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- $L_\lambda$ : The original intensity of the radiative beam
- $\sec\theta dz$ : The path length through the slab of the radiation.
- $k_\lambda^e$ : The mass extinction coefficient at wavelength  $\lambda$

Thus

$$dL_\lambda = -L_\lambda k_\lambda^e \rho \sec\theta dz, \quad (150)$$

where  $\rho$  is the density of the material.

## Mass extinction, absorption and scatter coefficients

For a specific material, it is useful to define the mass extinction  $k_{\lambda}^e$ . This specifies how strongly a unit mass of a medium depletes a direct beam of radiation of a given wavelength  $\lambda$  (or equivalently frequency  $\nu$ ) and has units  $\text{m}^2 \text{kg}^{-1}$ .

The mass extinction coefficient is the sum of the absorption  $k_{\lambda}^a$  and scattering  $k_{\lambda}^s$  coefficients.

$$k_{\lambda}^e = k_{\lambda}^a + k_{\lambda}^s \quad (151)$$

## Single scattering albedo

The **single scattering albedo** is defined as the ratio

$$\omega = \frac{k_{\lambda}^s}{k_{\lambda}^e} \quad (152)$$

The two extreme regimes are

- $\omega = 1$ : Pure scatter (conservative scatter)
- $\omega = 0$ : Pure absorption

We now integrate (150) over a finite depth slab to get the direct component of radiation as

$$L_{\lambda}^{z_1} = L_{\lambda}^{z_2} \exp \left( - \int_{z_1}^{z_2} k_{\lambda}^e \rho \sec \theta dz \right). \quad (153)$$

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### Optical Thickness/Depth

If from (153) we set

$$\delta_{\lambda} = \int_{z_1}^{z_2} k_{\lambda}^e \rho \sec \theta dz. \quad (154)$$

Then  $\delta_{\lambda}$  is known as the **optical thickness** or **optical depth** of the layer between  $z_1$  and  $z_2$ , essentially the integrated extinction coefficient.

Beer's (or Lambert's Law) then can be written

$$L_{\lambda}^{z_1} = L_{\lambda}^{z_2} e^{-\delta_{\lambda}} \quad (155)$$

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We lastly emphasize that the transmittance, absorptance, extinction and optical depth are **functions of wavelength** or equivalently frequency.

## Emission from slab

The emission by a slab in direction  $(\theta, \phi)$  is simply

$$L_{\lambda}^{\text{emitted}} = L_{\lambda} k_{\lambda}^a \rho \sec \theta dz \quad (157)$$

## Scattering from other directions

The scattering from other directions  $(\theta', \phi')$  is defined as

$$L_{\lambda}^{\text{scatter}} = \frac{\omega_{\lambda}}{4\pi} \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} L_{\lambda}(\theta', \phi') P(\theta', \phi', \theta, \phi) \sin\theta d\theta d\phi \quad (158)$$

We note

- The two integrals are to integrate over the sphere.

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- The two integrals are to integrate over the sphere.
- $4\pi$  is a normalization factor, the solid angle integrated over a sphere.
- Recall  $\sin\theta d\theta d\phi$  is the element of solid angle.

## Phase Function

$P(\theta', \phi', \theta, \phi)$  is called the **Phase Function** and gives the probability that a photon will be scattered from direction  $(\theta', \phi')$  into direction  $(\theta, \phi)$

For solar radiation ( $\lambda < 4\mu\text{m}$ ) **thermal emission** is unimportant as  $L_\lambda$  is very small.

For solar radiation ( $\lambda < 4\mu\text{m}$ ) **thermal emission** is unimportant as  $L_\lambda$  is very small. For thermal infrared ( $\lambda > 4\mu\text{m}$ ) scattering can generally be neglected, thus only the extinction and emission are important.

We now examine these in turn.

Gases are not like black bodies that absorb equally and completely at all wavelengths. Rather, they absorb only at specific, often narrow ranges of wavelengths.

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### Absorption by atmospheric gases

In general molecules possess three kinds of quantized energy

- Electronic Energy (Visible and UV)
- Vibrational Energy (Near IR, Thermal IR 0.7-20  $\mu\text{m}$ )
- Rotational Energy (Thermal IR)

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### Absorption by atmospheric gases

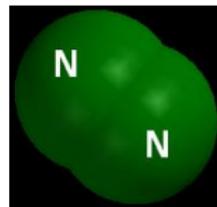
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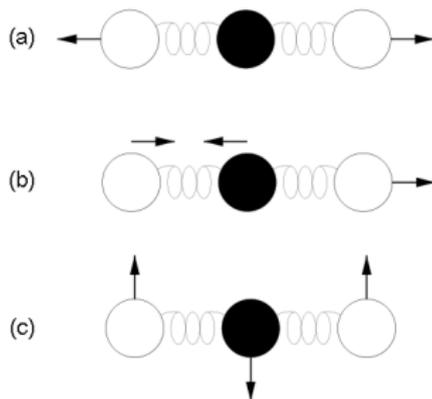
To absorb or emit radiation the molecule must possess a permanent dipole, as it vibrates or rotates the electric field varies, allowing the interaction with the incoming/outgoing radiation.

## $N_2$ and $O_2$

The major species of  $N_2$  and  $O_2$  are simple molecules that possess no permanent dipole (Fig. 98), and thus possess almost no vibrational or rotational absorption.



**Figure:**  $N_2$  molecule



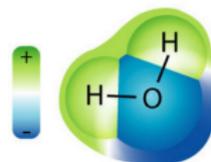
**Figure:**  $CO_2$  molecule with its main vibrational modes.

## $CO_2$

Carbon dioxide  $CO_2$  is a linear molecule, and thus from its symmetry possesses no dipole and thus no rotational spectrum. However, there are certain vibrational modes shown in (Fig. 99).

## H<sub>2</sub>O and O<sub>3</sub>

H<sub>2</sub>O and O<sub>3</sub> do possess a permanent dipole as the molecular structure is “bent” and hence asymmetric. They therefore also have a rotational spectrum. (Fig. 100),



**Figure:** H<sub>2</sub>O molecule and its charge structure

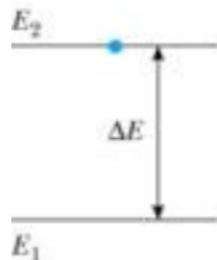
## Absorption Spectrum

The distribution of absorption lines with wavelength is very complicated. We have seen that the molecular states are defined by the molecule's modes of vibration and rotation and these modes are **quantized**, and may be excited by absorbing single photons (Fig. 101).

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The excited states do not persist: after some random amount of time, the molecules revert back to their original, lower energy state. The vibrational or rotational mode decays, emitting a photon.



**Figure:** Quantized vibrational energy levels.

## Absorption Spectrum

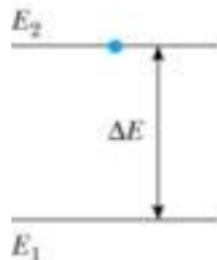
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The excited states do not persist: after some random amount of time, the molecules revert back to their original, lower energy state. The vibrational or rotational mode decays, emitting a photon.

The energy associated with with a transition between two states is

$$\Delta E = h\nu = h\tilde{\nu}c \quad (159)$$

where  $h$  is the Planck Constant and  $\tilde{\nu}$  is the **wavenumber** which is the reciprocal of the wavelength ( $\lambda^{-1}$ ).

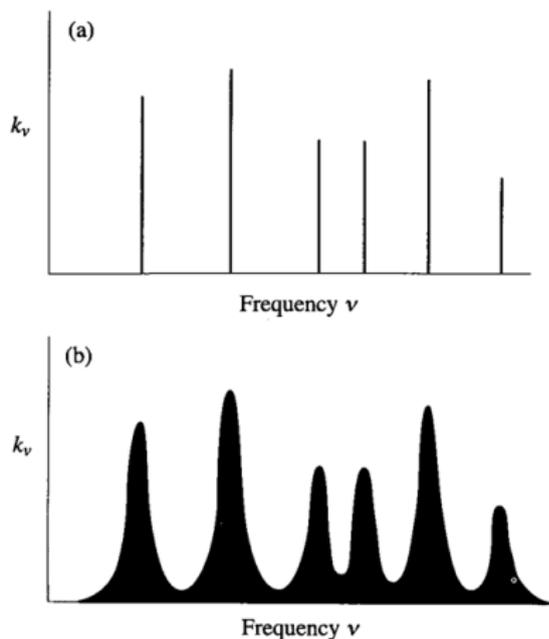


**Figure:** Quantized vibrational energy levels.

Due to the existence of various rotational and vibrational discrete levels, molecules can absorb or emit at many different wavelengths or wavenumbers, with the strength of the absorption related to the probability of the transition between the relevant energy levels.

## Line broadening

Due to the discretised nature of the molecular energy levels one might expect absorption and emission to occur at distinct set **delta function** wavelengths as in Fig. 102a. In reality the energy levels are slightly “fuzzy” due to **line broadening mechanisms** so that absorption is spread over a finite width of wavelengths (Fig. 102b).



**Fig. 3.5** Hypothetical line spectrum (a) before broadening, (b) after broadening.

**Figure:** (a) Hypothetical discrete absorption lines (b) actual absorption due to line broadening mechanisms

Three mechanisms for line broadening are

- **Natural Broadening:** a fundamental limit to the narrowness of the spectral lines due to Heisenberg's uncertainty principle, which relates the uncertainty of the time spent in excited states to the energy of the state transition

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**Where in the atmosphere will pressure broadening dominate?**

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Natural Broadening is overwhelmed by the other two processes. Q:

**Where in the atmosphere will pressure broadening dominate?**

Pressure broadening dominates in the troposphere where  $p$  is relatively high and Doppler broadening is important in the upper stratosphere ( $z > 30\text{km}$ ).

We recall from earlier (156) that the transmittance is related to optical depth as  $\tau_\lambda = e^{-\delta_\lambda} = \exp(-\int_{z_1}^{z_2} k_\lambda^e \rho \sec\theta dz)$ . If we assume a homogeneous material and a solar zenith angle of zero, then this simplifies to

$$\tau_\lambda = \exp(-k_\lambda^e \rho (z_2 - z_1)) \quad (160)$$

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$$\tau_\lambda = \exp(-k_\lambda^e \rho(z_2 - z_1)) \quad (160)$$

The absorption in a band depends on the product  $-k_\lambda^e \rho(z_2 - z_1)$ . If a particular gas is an efficient absorber (high  $-k_\lambda^e$ ) and/or is present in high amounts (large  $\rho(z_2 - z_1)$ ) then the incoming/outgoing radiation of a band of wavelengths around the line centre may be completely absorbed.

We thus find that we can have two regimes of **absorption lines** (**draw sketch**):

- **Weak Limit:** Absorption is linear with absorber amount
- **Strong Limit:** Radiation is totally absorbed at line centre, adding more absorber only increases absorption at band edges. Absorption increases proportionally to the **square root** of the absorber amount with the constant of proportionality depending on pressure.

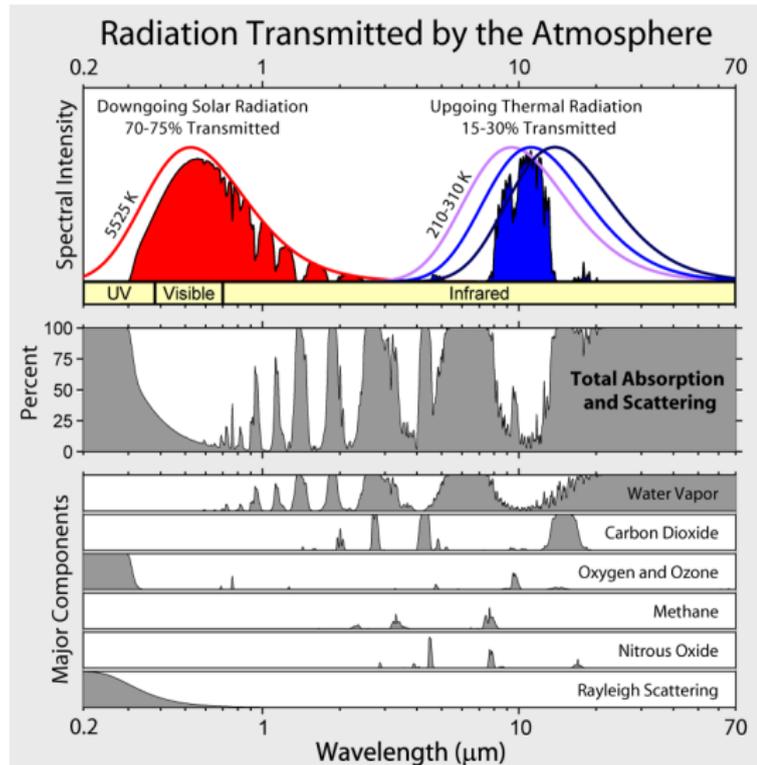
This is important when considering the greenhouse effect of gases. Carbon dioxide  $\text{CO}_2$  for example is present in substantially smaller amounts and in the strong limit. Doubling  $\text{CO}_2$  does **not** double its greenhouse effect.

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This is important when considering the greenhouse effect of gases. Carbon dioxide  $\text{CO}_2$  for example is present in substantially smaller amounts and in the strong limit. Doubling  $\text{CO}_2$  does **not** double its greenhouse effect. On the other hand CFCs are not present naturally and absorb in a part of the spectrum where other atmospheric gases are not active (referred to as an **absorption window**), and therefore their greenhouse effect increases linearly with concentration

The major absorption bands are shown in Fig. 103 with the contribution by the main active gases. It is seen that nearly all of the outgoing infra-red radiation from the earth's surface is absorbed by water vapour except for the window between 8 and approximately  $15\ \mu\text{m}$ . However,  $\text{CO}_2$  is strongly active in this window with an absorption band between  $14$  and  $16\ \mu\text{m}$ .

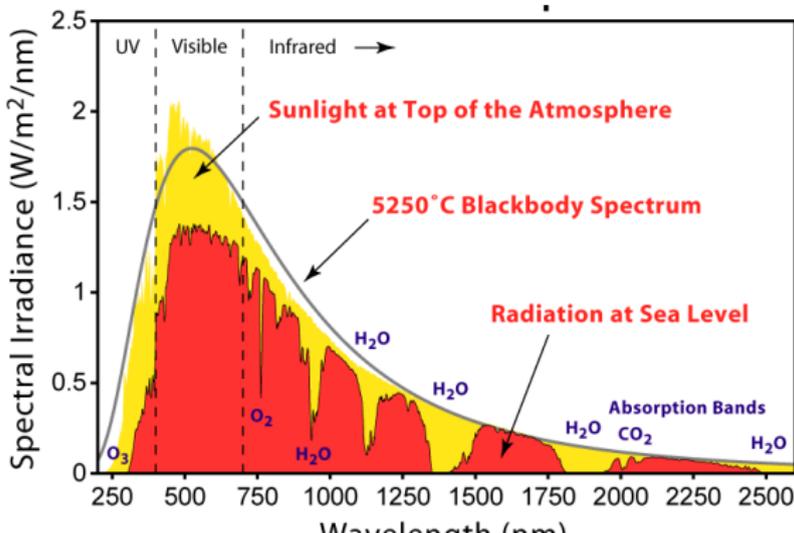


**Figure:** Upper panel shows the transmitted radiation in comparison to theoretical

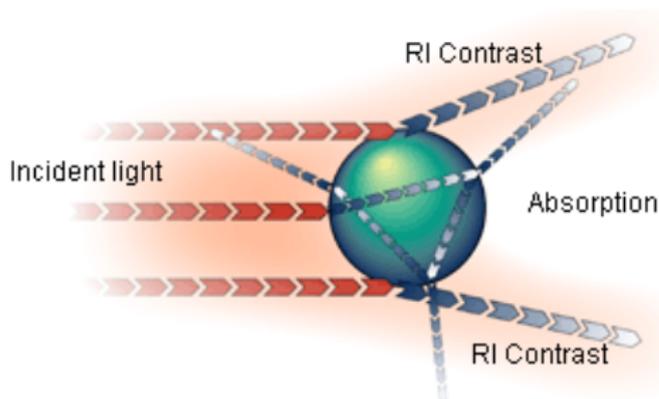
We saw in Fig. 103 the existence of the water vapour window in the IR. Q: In remote sensing what would this be useful for?

We saw in Fig. 103 the existence of the water vapour window in the IR. **Q: In remote sensing what would this be useful for?** The window region of 8-13  $\mu\text{m}$  is useful for remote sensing of the surface (e.g. SST).

In contrast to the earth's infra-red radiation, the majority of the solar radiation at the top of atmosphere reaches the earth's surface, with the atmosphere on average absorbing around 17% of solar radiation. We supplement Fig. 103 with Fig. 104, which has the specific gas absorption band marked and also gives the exact incoming solar radiation spectra in addition to the theoretical blackbody curve.



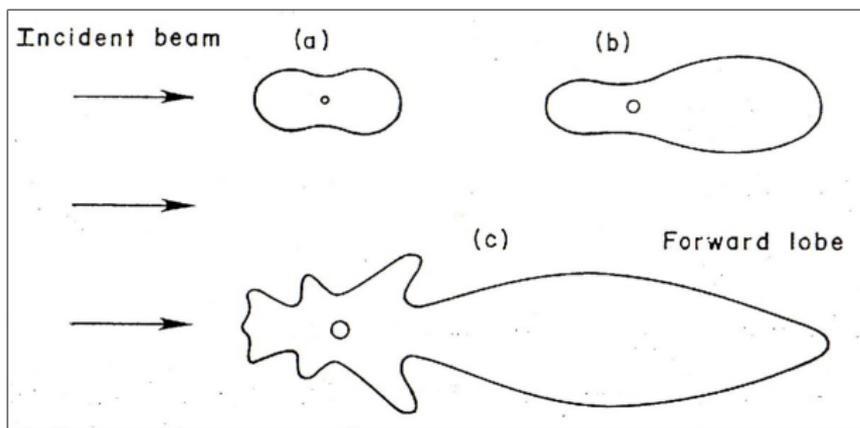
The scattering of radiation by a particle, depends strongly on the ratio of the wavelength of the radiation to the radius of the particle. The theory that governs this is called Mie theory, involving the internal and external refraction of the radiation by the particle and the full description is beyond the scope of this course. (Fig.fig:mie)



**Figure:** Schematic of mie scattering (source [www.malvern.co.uk](http://www.malvern.co.uk))

The scattering regime has a large impact on the phase function  $P(\theta', \phi', \theta, \phi)$  (Fig. 106, see also Maxwell animations online). The relevant parameter to determine the scattering regime is the ratio of the particle circumference to the radiation wavelength  $2\pi r/\lambda$ .

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**Figure:** Scatter regimes as a function of  $2\pi r/\lambda$  (a)  $2\pi r/\lambda \ll 1$ , (b)  $2\pi r/\lambda \sim 1$  (c)  $2\pi r/\lambda > 1$

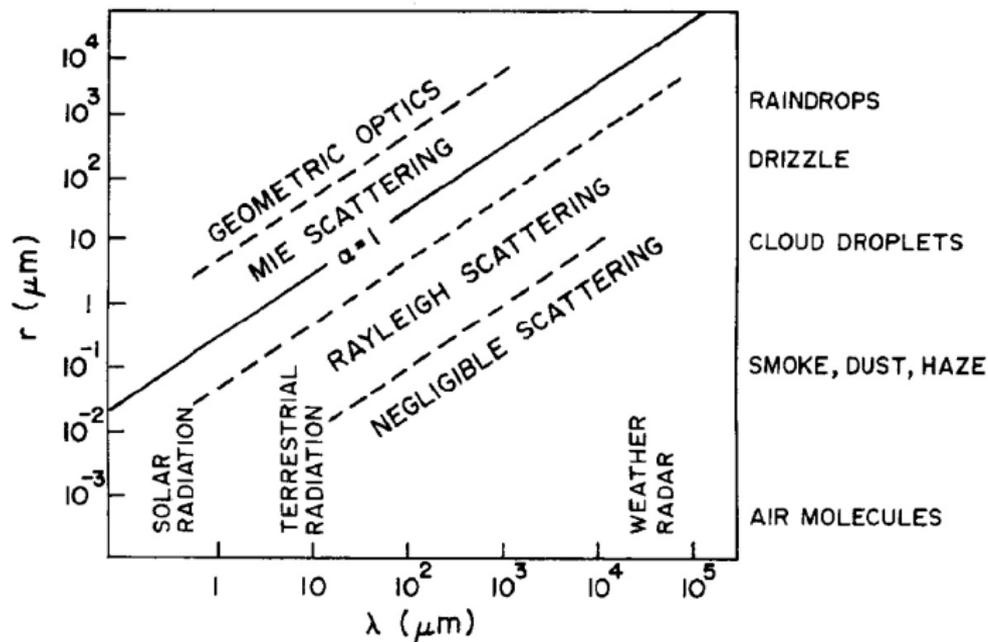
One can see that in the regime  $2\pi r/\lambda \ll 1$  the scattering is more regular in direction, with the phase function losing its directional dependence in the limit  $2\pi r/\lambda \rightarrow 0$ .

## Rayleigh Scattering

If we assume in the regime  $2\pi r/\lambda \ll 1$  (the limit is about  $5 \times 10^{-3}$ ) that phase function is independent of direction and the scattering is thus a function of  $\lambda$  only then this approximation is called **Rayleigh Scattering**. In this regime the scatterer can be treated as a single dipole.

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**Figure:** Scattering regimes as a function of wavelength and particle radius.

In general the radiatively “active” species are the trace gases such as  $\text{CO}_2$ ,  $\text{H}_2\text{O}$ ,  $\text{O}_3$ . Rayleigh Scattering is one of the few examples where the major species such as  $\text{N}_2$  and  $\text{O}_2$  are “active”. Q: why is the sky blue?

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The scattering extinction coefficient in the Rayleigh regime is proportional to the inverse fourth power of the wavelength,  $k_\lambda^s \propto \frac{1}{\lambda^4}$ . Red light has  $\lambda = 0.7\mu\text{m}$  while blue light has  $\lambda = 0.4\mu\text{m}$ , indicating an order of magnitude increase in scattering at blue wavelengths.

## Contribution of Rayleigh scatter to planetary albedo

Rayleigh scattering's contribution to albedo depends on the solar zenith angle. Q: why?.

## Contribution of Rayleigh scatter to planetary albedo

Rayleigh scattering's contribution to albedo depends on the solar zenith angle. Q: why?. The solar zenith angle determines the pathlength of the radiation. When the sun is overhead Rayleigh scattering contributes 0.037 to the albedo, while the figure is 0.13 for a solar zenith angle of  $80^\circ$ . The global average value is about 0.06, thus  $0.06/0.30 = 20\%$  of the planetary albedo is due to Rayleigh scatter

## Scattering and absorption by clouds

In clouds Rayleigh scattering approximation no longer holds and  $2\pi r/\lambda \approx 100$ , and the full (complicated) Mie theory is required.

We saw in Fig. 106 that in this case there was a strong “forward lobe” where the forward scatter was much stronger than the backwards scatter. For liquid water droplets the forward scattering lobe can be many orders of magnitudes greater than the back-scattered radiation.

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We saw in Fig. 106 that in this case there was a strong “forward lobe” where the forward scatter was much stronger than the backwards scatter. For liquid water droplets the forward scattering lobe can be many orders of magnitudes greater than the back-scattered radiation. This leads to an apparent paradox as clouds reflect up to 70% of the incident solar radiation back to space. **Q: how can this occur?**

The solution is that we have so far considered only single scattering events. In optically thin clouds photons may undergo only a single scattering event and thus the vast majority pass through unimpeded. In optically thick clouds though multiple scattering may occur (**draw sketch**). Thus the possibility of light reemerging at the cloud top is much higher. **This is why thick clouds appear dark when viewed from the surface**. Cloud albedo asymptotes to around 0.8 for thick clouds.

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We recall that  $\omega$  tells us according to (152) what proportion of the direct beam extinction is due to scatter or absorption. For solar radiation  $\omega > 0.9$  meaning most of the radiation is scattered and indeed for visible wavelengths  $\omega > 0.999$ ; the **conservative scattering** regime.

However, in the thermal infra-red generally  $\omega < 0.5$  implying that there is little chance of a photon at these wavelength being multiply scattered and escaping from the cloud. Thus **clouds are essentially black bodies in the infra-red.**

We can see from the above that clouds will have large effect on the earth's radiation budget depending on their height and optical depth.

### Low clouds

- SW: Low clouds with sufficient optical depth will reflect sunlight back to space efficiently, thereby reducing the amount of solar radiation absorbed at the surface.
- LW:

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- LW: In the long wave infra red, clouds absorb the surface emitted radiation efficiently as they are effectively black bodies. They emit infrared radiation back to earth at the approximate temperature of the cloud lower surface, and to space at the cloud top temperature. **Q: Do low clouds have a strong impact in the IR therefore?** The answer is no, since the cloud top temperature will be similar to the surface.

Combining these two effects implies that low clouds cool the earth's surface.

## High clouds

High clouds of both reflect light back to space, but they can also have a strong IR signature since they are very cold (temperatures close to that of the tropopause). The balance can be in favour of warming or cooling depending on the crystal type<sup>a</sup> and the optical depth.

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<sup>a</sup>Remember that these are ice clouds with complex crystal geometries affecting scattering properties. See Macke et al. (1996) for a seminal analysis of ice crystal scattering.

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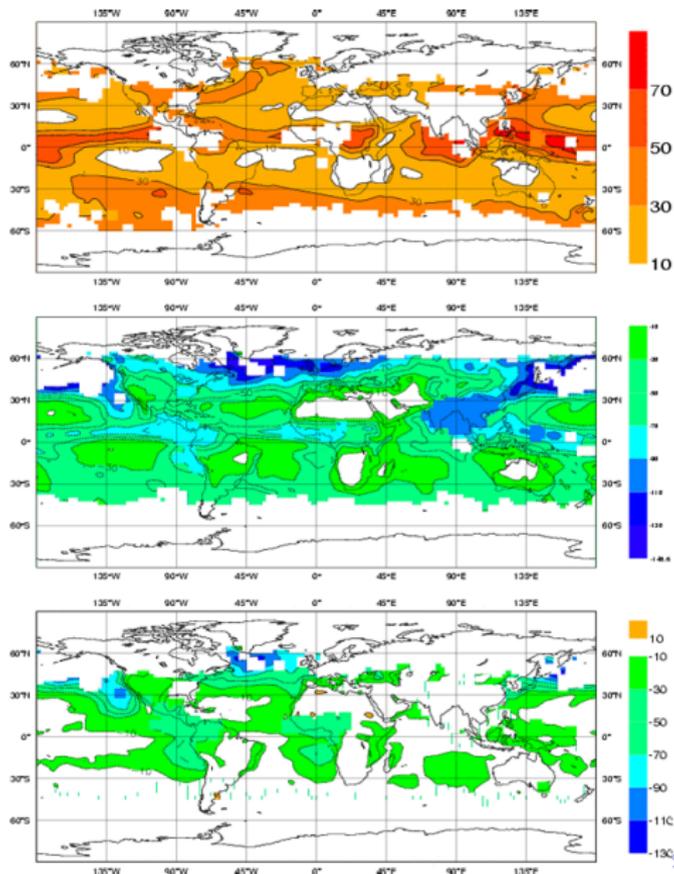
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This may be a coincidence of the present climate and not hold in past or future climates, but some authors have suggested that this result is no accident, and have put forward mechanisms involved negative feedbacks between radiation and convection that maintain this balance (see Hartmann et al., 2001).

These effects of clouds can be seen in Fig. 108 which shows the cloud radiative forcing in the SW, IR and net (SW+IR). These are calculated at each location by comparing the mean TOA outgoing radiative flux **at all times** with the mean averaged for only those times **when clouds are present**. The difference between these two values gives a measure of the cloud radiative forcing.

The effect of the low cloud (stratocumulus) regimes off the West coasts of the major continents in clear in the SW but they are not visible in LW. In contrast the deep convective clouds in the tropics have a marked and opposing signal in both the IR and SW.



## Calculating heating rates

To calculate the atmospheric heating rate a range of models are used that divide the spectrum up into a number of bands, with the spectrum resolution depending on available computing time.

- **Line by Line:** The most complicated models will divide the spectrum of  $0\text{-}2500\text{cm}^{-1}$  into well over 100,000 spectral intervals. These models are very expensive to run but provide the benchmarks by which to test the cheaper models below.
- **Narrow Band:** These models will divide the infra-red spectrum into  $\sim 250$  spectral intervals, but these models are still too expensive to use in weather prediction or climate models.
- **Broad Band:** These may employ 5 to 20 spectral intervals, although numerical tricks can be used to improve representation of gas absorption bands.

The actual heating rate will depend on the amount of the gas absorber present and the strengths of the absorption by the gas.

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$$\frac{dT}{dt} = \frac{\Delta E}{c_p\rho\Delta z}. \quad (161)$$

with the units of the RHS being  $\text{K s}^{-1}$  as expected.

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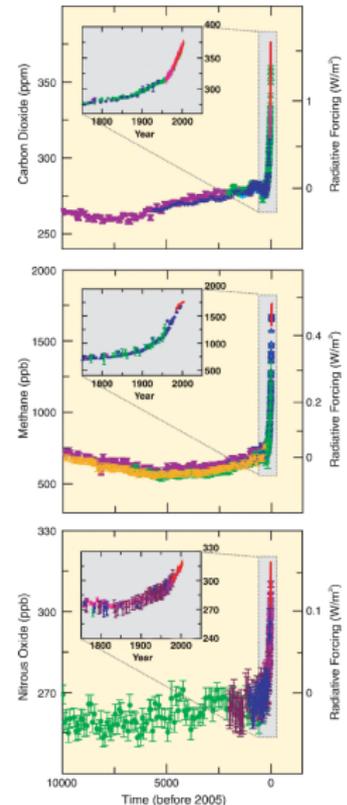
with the units of the RHS being  $\text{K s}^{-1}$  as expected. We recall that the main absorbers in the solar spectrum are  $\text{O}_3$  and  $\text{H}_2\text{O}$ , with ozone absorbing mainly in the stratosphere resulting in a heating rate of  $10\text{K day}^{-1}$  at a height of 50km, and water vapour mainly active in the troposphere (Q: why?) with a heating rate of approximately  $1\text{K day}^{-1}$ .

In the thermal infra-red calculation the situation is more complicated due to the emission in each layer. Infrared radiation tends to cool the atmosphere, with the dominant coolers being  $\text{H}_2\text{O}$  in the troposphere and  $\text{CO}_2$  in the stratosphere.

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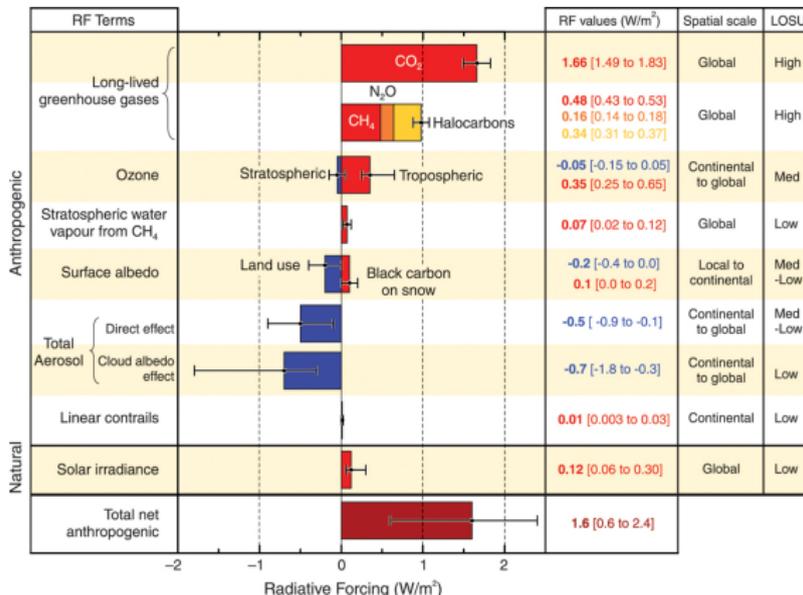
## Anthropogenic global warming

From the above discussion it is clear that increasing the concentrations of radiatively active gases that absorb in the infra-red will tend to warm the surface. Thus burning fossil fuels increases concentrations of  $\text{CO}_2$  which to date are estimated to have had an equivalent TOA radiative forcing of around  $1.5 \text{ W m}^{-2}$  (Fig. 109).



Likewise, humans can have other climate impacts such as changing surface albedo through land-use changes or changing atmospheric albedo through the emission of aerosols that reflect sunlight (Fig. 110).

Likewise, humans can have other climate impacts such as changing surface albedo through land-use changes or changing atmospheric albedo through the emission of aerosols that reflect sunlight (Fig. 110). Indeed it is the later effect that is now thought to be partly the cause of the cooling in northern latitudes during the 1950s and 1960s.



The calculation of the projected man-induced warming in future is greatly complicated by the existence of natural feedbacks, that may **amplify** or **reduce** the eventual warming. Two of the major positive feedbacks are:

- **Ice Albedo Feedback**: Increased temperatures → ice melting at poles → lower albedo → increased solar absorption → increased temperatures

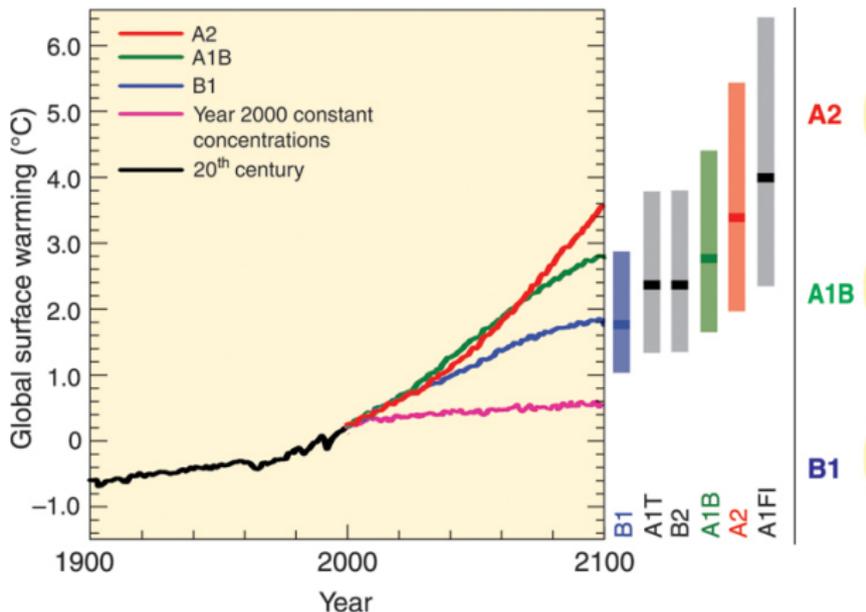
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- **Ice Albedo Feedback:** Increased temperatures → ice melting at poles → lower albedo → increased solar absorption → increased temperatures
- **Water Vapour Feedback:** Increased temperatures → more water vapour → enhanced greenhouse effect → increased temperatures

One of the major uncertainties though is the future role played by clouds. This is complicated as clouds could get thicker, thinner, higher, change phases, change radiative properties, etc. Global climate models do not agree on the role of clouds and many models show clouds amplifying the future warming, while other show clouds offsetting future possible warming. Note: Just because clouds tend to **cool** the present-day climate does not imply that they will **offset** future warming.

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Despite these uncertainties, estimates of future warming are made and must also take into account uncertainties in emission scenarios. Projections from IPCC AR4 are shown in Fig. 111.



**Figure:** Future projected warming from many different climate models under different emission scenarios (source IPCC AR4, see this report for

The end!

## Readings

The following reading assignments are to be presented in week 5 of the course.

Each student will make a 20 minute presentation and then will field questions for 10 minutes from the other students and myself. The reading will contribute 30% to the final mark.

- 1 Buoyancy sorting model of convection** Explain this method of representing the convective process: (Paluch, 1979; Raymond and Blyth, 1986)
- 2 Mass flux convection schemes** Explain this method of representing the convective process as presented by Tiedtke (1989)
- 3 Adjustment convection schemes** Explain this method of representing the convective process as presented by Betts (1986); Betts and Miller (1986)
- 4 Thermodynamic control of tropical rainfall** Explain the ideas of Raymond (2000)
- 5 Convective equilibrium in the tropics** Explain the boundary layer equilibrium ideas of Raymond (1995)
- 6 Squall Lines** What is a squall line and how does it propagate according to (Rotunno et al., 1988)?
- 7 CAPE/CIN** Explain the concept of CAPE and CIN and the findings of Parker (2002) concerning CAPE and CIN variations
- 8 Saturation point analysis** Explain the core points of the Betts (1982) saturation point analysis.
- 9 Convective boundary layer** Explain the model of convective equilibrium of Betts and Ridgway (1980)

- Berry, E. X., 1967: Cloud droplet growth by collection. *J. Atmos. Sci.*, **24 (6)**, 688–701.
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- Betts, A., 1982: Saturation point analysis of moist convective overturning. *J. Atmos. Sci.*, **39 (7)**, 1484–1505.
- betts, A. K., 1986: A new convective adjustment scheme. part 1: Observation and theoretical basis. *Q. J. R. Meteorol. Soc.*, **112 (473)**, 677–691.
- Betts, A. K. and M. J. Miller, 1986: A new convective adjustment scheme. Part 2: Single column tests using GATE WAVE, BOMEX, ATEX and arctic air-mass data sets. *Q. J. R. Meteorol. Soc.*, **112**, 693–709.
- Betts, A. K. and W. Ridgway, 1989: Climatic Equilibrium of the Atmospheric Convective Boundary Layer over a Tropical Ocean. *J. Atmos. Sci.*, **46 (17)**, 2621–2641.
- Bjerknes, H. R. and R. R. Brabham, Jr., 1948: Thunderstorm structure